ELETRICAL ANALOGY MODELING FOR ONE-DIMENSIONAL ABLATION PROBLEM

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Abstract

The transient heat transfer in a solid undergoing ablation is a nonlinear problem, which involves a moving boundary that is not known a priori. In this paper, the ablation problem is solved using constant material properties and constant heat flux. The analogy between the heat transfer in a solid body and the current in an electric circuit for time-dependent electrical devices is used. The results compared quite well with the numerical solution presented by Blackwell.

Nomenclature

C	Global Electric Capacitance
c_p	Specific Heat at Constant Pressure
F	Electric Power Source
f	Mean Temperature Position
L	Electric Impedance
k	Thermal Conductivity
q	Heat Flux
$1/R$	Electric Conductance
$\overline{T}$	Mean Temperature / Mean Electric Potential
T_r	Initial Temperature / Reference Electric Potential
T_1	Front Face Temperature / Front Face Electric Potential
T_2	Heat Penetration Front Temperature / Heat Penetration Front Position Electric Potential
T_m	Melting Temperature
t	Time Coordinate
t_m	Melting Time
x	Space Coordinate
d_1	Heat Penetration Front Position
d_2	Ablation Front Position
r	Density
l	Heat of Ablation

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Introduction

Transient heat conduction in a solid undergoing ablation represents an area of great technological importance. Problems of this type are inherently nonlinear and involve a moving boundary that is not known a priori. According Chung[1] and Zien[2], the exact analytical solution for transient heat transfer in a solid accompanied by ablation is very difficult and practically nonexistent. Only numerical and approximate analytical solutions have been made available and they necessarily require considerable numerical computation, even if a simplified model of the problem is used in the study.

This work makes use of the similarity of the mathematical formulation of heat transfer in a solid body and the carrying of electric current in an electric circuit, as presented by Horvay[3] to the freezing of a growing liquid column process. This technique represents a different method to solve the phase-change ablation problem.

Literature Review

Landau[4] first proposed the idealized ablation problem and solved it by numerical integration for the case of a semi-infinite melting solid with constant properties and with its face heated at a constant rate.

Sunderland and Grosh[5] solved the same Landau's[4] problem, but they used the finite difference method of solution for the case where the heat flux at the face may vary with the time.

Goodman[6] studied Landau's problem using the heat balance integral method. Biot and Agrawal[7] used the variational method for the analysis of ablation for variable properties.

Storti[8] considered a one-phase ablation problem as it was a two-phase Stefan one, by the introduction of a fictious phase occupying the region where the material has been removed. He solved it by the finite element method.

Physical Model

Storti[8] considered that, when a severe radiation and/or convection heat flux reaches one of the faces of the ablating material, initially in its ‘virgin’ phase, the temperature rises, and the material can experience one or several chemical reactions, which must be strongly endothermic, for the ablate phenomena to be effective. The material exposed to the thermal load is removed by mechanical (high shear stresses) or chemical action. In the case of a phase-change to a phase with very low mechanical strength, the material is considered removed immediately after it reaches the phase-change temperature. This Stefan-type or phase-change ablation model is the physical model adopted in this work.

Blackwell[9] used the finite control volume method with exponential differential to solve Landau’s problem and his results will be used as a benchmark in this work.

The following physical model is adopted: a semi-infinite ablative material is heated by an uniform and constant heat source. In the beginning, the heat penetrates the material, raising the temperature of part of the material. The length of this part is named $d_2(t)$, where $d_2(0)=0$. The heating continues until the front face temperature (T_1) reaches the melting temperature (T_m) and the ablation starts. During the ablation, part of the heat is used to keep T_1 at T_m and the rest is used to change the phase of the ablation material. The phase-change phenomenon consumes part of the virgin material. The length of this part is denominated $d_1(t)$, where $d_1(t_m)=0$, where t_m is the time in which T_1 reaches T_m. Fig. 1 shows a schematic of this physical model.

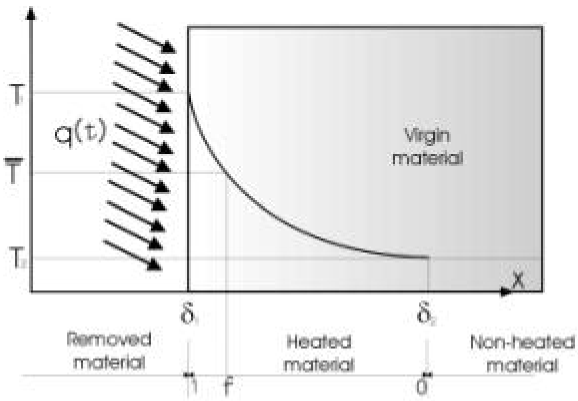

Fig. 1 - Physical model.

Analytical Model

The following one-dimensional heat transfer equation is used to determine the ablation rate and the heat penetration depth:

$$r\, c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x} \right) \tag{1}$$

This equation is integrated in x from $d_1(t)$ to $d_2(t)$. The results are rearranged using Leibnitz’s integral formula, getting:

$$\begin{aligned} & r\, c_p \frac{d}{d t} \int_{d_1(t)}^{d_2(t)} T\, dx - r\, c_p\, T\left(d_2(t)\right) \frac{d\, d_2(t)}{d t} \\ & + r\, c_p\, T\left(d_1(t)\right) \frac{d\, d_1(t)}{d t} \\ & = k \frac{\partial T\left(d_2(t)\right)}{\partial x} - k \frac{\partial T\left(d_1(t)\right)}{\partial x} \end{aligned} \tag{2}$$

Defining $\overline{T} = \dfrac{\int_{d_1(t)}^{d_2(t)} T\, dx}{\left(d_2(t) - d_1(t)\right)}$, substituting in Eq. 2, simplifying and rearranging, it is obtained:

$$r\, c_p \left(d_2(t) - d_1(t)\right) \frac{d \overline{T}}{d t} = r\, c_p \frac{d d_2(t)}{d t} \left(T\left(d_2(t)\right) - \overline{T}\right) + r\, c_p \frac{d d_1(t)}{d t} \left(\overline{T} - T\left(d_1(t)\right)\right) + k \left[\frac{\partial T\left(d_2(t)\right)}{\partial x} - \frac{\partial T\left(d_1(t)\right)}{\partial x} \right] \tag{3}$$

Using finite difference technique to discretize the space derivates, one gets:

$$\frac{\partial T(d_2(t))}{\partial x} = \frac{T(d_2(t)) - \overline{T}}{f(d_2(t) - d_1(t))}$$
$$\frac{\partial T(d_1(t))}{\partial x} = \frac{\overline{T} - T(d_1(t))}{(1-f)(d_2(t) - d_1(t))}$$

where f is a constant value between 0 and 1 that represents the position of the mean temperature value (see Fig. 1). Substituting these expressions in Eq. 3 and collecting similar terms, it is obtained:

$$\begin{aligned} & r\, c_p (d_2(t) - d_1(t)) \frac{d\overline{T}}{dt} \\ & = \left[r\, c_p \frac{dd_2(t)}{dt} + \frac{k}{f(d_2(t) - d_1(t))} \right] \\ & \quad (T(d_2(t)) - \overline{T}) \\ & + \left[r\, c_p \frac{dd_1(t)}{dt} - \frac{k}{(1-f)(d_2(t) - d_1(t))} \right] \\ & \quad (\overline{T} - T(d_1(t))) \end{aligned} \tag{4}$$

The terms of Eq. 4 can be associated with the following components of an electrical circuit (see Fig. 2):

Table 1 – Electric Analogy Parameters

$r\, c_p (d_2(t) - d_1(t)) = C(t)$	Global electric capacitance
$r\, c_p \frac{dd_2(t)}{dt} = F_2(t)$	Electric power source
$\frac{k}{f(d_2(t) - d_1(t))} = \frac{1}{R_2(t)}$	Electric conductance
$r\, c_p \frac{dd_1(t)}{dt} = F_1(t)$	Electric power source
$\frac{k}{(1-f)(d_2(t) - d_1(t))} = \frac{1}{R_1(t)}$	Electric conductance
$T(d_2(t)) = T_2$	Electric potential
$T(d_1(t)) = T_1$	Electric potential
$L(t) = r\, l \frac{dd_1}{dt}$	Electric impedance

Substituting these terms in Eq. 4, one gets:

$$\begin{aligned} & C(t) \frac{d\overline{T}}{dt} = \left[F_2(t) + \frac{1}{R_2(t)} \right] (T_2 - \overline{T}) \\ & + \left[F_1(t) - \frac{1}{R_1(t)} \right] (\overline{T} - T_1) \end{aligned} \tag{5}$$

The last equation is a well know expression for transient behavior of electric circuits and represents the circuit shown in Fig.2

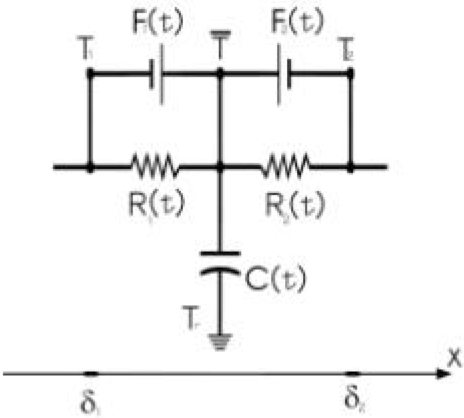

Fig. 2 - Electrical circuit represented by Eq. 5

The following boundary conditions can be considered for the pre-ablation period:

$$\frac{dd_1}{dt} = 0 \tag{6}$$

$$-k \frac{\partial T}{\partial x} = q(t) \qquad , x = d_1 \tag{7}$$

$$-k \frac{\partial T}{\partial x} = r\, c_p \frac{dd_2(t)}{dt} \Delta T \qquad , x = d_2 \tag{8}$$

$$T_2 = T_r \qquad , x = d_2 \tag{9}$$

Using finite difference technique to discretize the space derivate of the boundary conditions and associating it with the electrical components, Eqs. 5 to 9 can be rewritten as:

$$\frac{dd_1}{dt} = \frac{F_1(t)}{r\, c_p} = 0 \tag{10}$$

$$-k \frac{\partial T(d_1(t))}{\partial x} = \frac{(T_1 - \overline{T})}{R_1(t)} = q(t) \tag{11}$$

$$-k\frac{\partial T}{\partial x} = -k\frac{\left(T_2-\overline{T}\right)}{f\left(d_2(t)-d_1(t)\right)} = \frac{\left(\overline{T}-T_2\right)}{R_2(t)} = r\,c_p\frac{d\,d_2(t)}{d\,t}\left(\overline{T}-T_2\right) = F_2\left(\overline{T}-T_2\right) \qquad (12)$$

$$T_2 = T_r \qquad (13)$$

These boundary conditions, when applied to Eq. 5, lead to the simplified electric circuit show in Fig. 3.

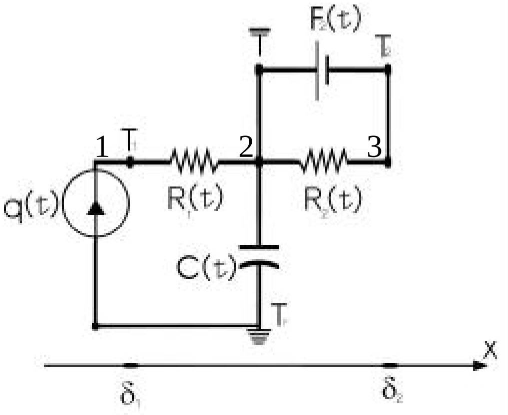

Fig. 3 - Electrical circuit represented by Eq. 5 with the pre-ablation boundary condictions.

Similarly, the following boundary conditions can be considered for the ablation period:

$$-k\frac{\partial T}{\partial x} = q - r\ l\,\frac{d\,d_1}{d\,t} \quad ,\ x = d_1 \qquad (14)$$

$$T_1 = T_m \quad ,\ x = d_1 \qquad (15)$$

$$-k\frac{\partial T}{\partial x} = r\,c_p\frac{d\,d_2(t)}{d\,t}\Delta T \quad ,\ x = d_2 \qquad (16)$$

$$T_2 = T_r \quad ,\ x = d_2 \qquad (17)$$

Adopting the same procedure as before, i.e., using finite difference technique to discretize the space derivate of the boundary conditions and associating it to the electrical components, one gets:

$$-k\frac{\partial T\left(d_1(t)\right)}{\partial x} = \frac{\left(T_1-\overline{T}\right)}{R_1(t)} = q(t) - L(t) \qquad (18)$$

$$T_1 = T_m \qquad (19)$$

$$-k\frac{\partial T}{\partial x} = -k\frac{\left(T_2-\overline{T}\right)}{f\left(d_2(t)-d_1(t)\right)} = \frac{\left(\overline{T}-T_2\right)}{R_2(t)} = r\,c_p\frac{d\,d_2(t)}{d\,t}\left(\overline{T}-T_2\right) = F_2\left(\overline{T}-T_2\right) \qquad (20)$$

$$T_2 = T_r \qquad (21)$$

In these equations, $L(t)$ is the electric impedance defined as $L(t) = r\ l\,\frac{d\,d_1}{d\,t}$ (see Table 1).

Applying these boundary conditions to Eq. (5), one gets the simplified electrical circuit shown in Fig.4.

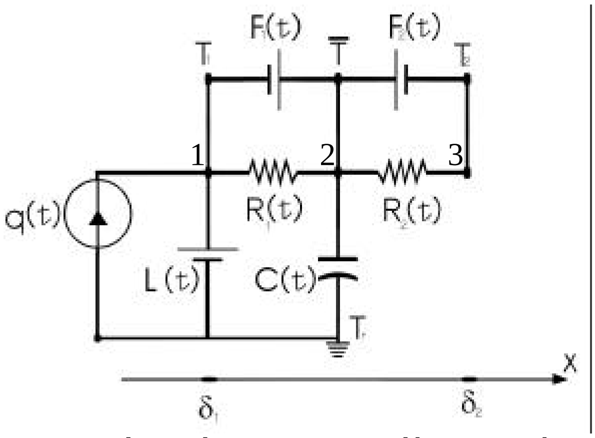

Fig. 4 - Electrical circuit represented by Eq. 5 with the ablation boundary conditions.

The electrical circuits shown in Fig.3 and Fig.4 can be solved by the Kirchoff Nodes Law, which states that the sum of all electric current that gets in or out of a node is equal to zero. The application of this law leads to the following equations, concerning the electrical circuit shown in Fig. 3, for the pre-ablation period:

Node 1: $$\frac{\left(T_1-\overline{T}\right)}{R_1(t)} = q(t) \qquad (22)$$

Node 2: $$C(t)\frac{d\overline{T}}{d\,t} = \left[F_2(t) + \frac{1}{R_2(t)}\right]\left(T_2-\overline{T}\right) - \frac{\left(\overline{T}-T_1\right)}{R_1(t)} \qquad (23)$$

Node 3: $$F_2(t) = \frac{1}{R_2(t)} \qquad (24)$$

Also, the following equation can be used:

$$T_2 = T_r \qquad (25)$$

Substituting Eq. 22, 24 and 25 in Eq. 23 and simplifying, one gets for Node 2:

$$\frac{d\overline{T}}{d\,t} = \frac{2F_2(t)}{C(t)}\left(T_r-\overline{T}\right) + \frac{q(t)}{C(t)} \qquad (26)$$

Substituting the electric components by their associated functions in the Eq. 22, 24 and 26 and manipulating the expressions, one gets:

$$T_1 = \frac{(1-f)(d_2(t)-d_1(t))}{k}\, q(t) + \overline{T} \tag{27}$$

$$\frac{d\overline{T}}{dt} + \frac{2\overline{T}}{(d_2(t)-d_1(t))}\frac{dd_2(t)}{dt} = \frac{2T_r}{(d_2(t)-d_1(t))}\frac{dd_2(t)}{dt} + \frac{q(t)}{r\,c_p(d_2(t)-d_1(t))} \tag{28}$$

$$r\,c_p\frac{dd_2(t)}{dt} = \frac{k}{f(d_2(t)-d_1(t))} \tag{29}$$

Defining:

$$u(t) = d_2(t) - d_1(t) \tag{30}$$

and substituting on the expressions above, it is obtained respectively:

$$T_1 = \frac{(1-f)\,u(t)}{k}\, q(t) + \overline{T} \tag{31}$$

$$\frac{d\overline{T}}{dt} + \frac{2\overline{T}}{u(t)}\frac{du(t)}{dt} = \frac{2T_r}{u(t)}\frac{du(t)}{dt} + \frac{q(t)}{r\,c_p\,u(t)} \tag{32}$$

$$\frac{du(t)}{dt} = \frac{k}{f\,r\,c_p\,u(t)} \tag{33}$$

Solving the Eq. 33 for $u(t_o) = 0$ as initial condition one gets:

$$u(t) = \sqrt{\frac{2\,k\,t}{f\;r\;c_p}} \tag{34}$$

Substituting this expression on Eq. 32 and simplifying one gets:

$$t\frac{d\overline{T}}{dt} + \overline{T} = T_r + q(t)\sqrt{\frac{f\,t}{2\,r\,c_p\,k}} \tag{35}$$

Using $\overline{T}(t_o) = T_r$ as the initial condition for the solution of Eq. 35, the following expression arises:

$$\overline{T} = T_r + \frac{1}{t}\int_0^t q(t)\sqrt{\frac{f\,t}{2\,r\,c_p\,k}}\,dt \tag{36}$$

Substituting Eq. 34 and Eq. 36 at the Eq. 31, the following expression is obtained:

$$T_1 = (1-f)\sqrt{\frac{2t}{k\,f\,r\,c_p}}\,q(t) + T_r + \frac{1}{t}\int_0^t q(t)\sqrt{\frac{f\,t}{2\,r\,c_p\,k}}\,dt \tag{37}$$

Substituting the Eq. 34 into Eq. 30 and rearranging the terms, it is obtained:

$$d_2(t) = \sqrt{\frac{2\,k\,t}{f\;r\,c_p}} + d_1(t) \tag{38}$$

The end of pre-ablation period is defined by $T_1 = T_m$, at $t = t_m$. In the case of a constant heat flux $(q(t) = q)$, t_m can be calculated by Eq. 37, giving:

$$t_m = \frac{f\,r\,c_p\,k}{2}\left(\frac{T_m - T_r}{\left(1-\frac{2f}{3}\right)q}\right)^2 \tag{39}$$

Using the Kirchoff Law for the ablation period electrical circuit shown in Fig. 4, one gets the following equations:

Extra Equation: $T_1 = T_m$ (40)

Extra Equation: $T_2 = T_r$ (41)

Node 1: $-\dfrac{1}{R_1(t)}\left(\overline{T} - T_1\right) = q(t) - L(t)$ (42)

Node 2: $$\frac{d\overline{T}}{dt} = \left[\frac{F_2(t)}{C(t)} + \frac{1}{C(t)R_2(t)}\right]\left(T_2 - \overline{T}\right) + \left[\frac{F_1(t)}{C(t)} - \frac{1}{C(t)R_1(t)}\right]\left(\overline{T} - T_1\right) \tag{43}$$

Node 3: $F_2(t) = \dfrac{1}{R_2(t)}$ (44)

Substituting Eq. 40, 41, 42 and 44 at Eq. 43 and simplifying the terms, one gets for Node 2,

$$\frac{d\overline{T}}{dt} = \frac{2F_2(t)}{C(t)}\left(T_r - \overline{T}\right) + \frac{F_1(t)}{C(t)}\left(\overline{T} - T_m\right) + \frac{q(t)}{C(t)} - \frac{L(t)}{C(t)} \tag{45}$$

Substituting the electrical components by their associated functions in Eqs. 42, 44 and 45 and manipulating these expressions, one gets:

$$\frac{dd_1(t)}{dt} = \frac{k\left(\overline{T} - T_m\right)}{r\;l\,(1-f)(d_2(t)-d_1(t))} + \frac{q(t)}{r\;l} \tag{46}$$

$$\frac{d\overline{T}}{dt}=\frac{2\left(T_r-\overline{T}\right)}{\left(d_2(t)-d_1(t)\right)}\frac{dd_2(t)}{dt}+\frac{q(t)}{r\,c_p\left(d_2(t)-d_1(t)\right)}+\frac{1}{\left(d_2(t)-d_1(t)\right)}\left(\left(\overline{T}-T_m\right)-\frac{l}{c_p}\right)\frac{dd_1}{dt} \qquad (47)$$

$$\frac{dd_2(t)}{dt}=\frac{k}{r\,c_p\,f\left(d_2(t)-d_1(t)\right)} \qquad (48)$$

Combining the Eq. 48 and Eq. 46 one gets:

$$\frac{d\left(d_2(t)-d_1(t)\right)}{dt}=-\frac{k\left(\overline{T}-T_m\right)}{r\,l\,(1-f)\left(d_2(t)-d_1(t)\right)}-\frac{q(t)}{r\,l}+\frac{k}{r\,c_p\,f\left(d_2(t)-d_1(t)\right)} \qquad (49)$$

Using again the $u(t)$ definition, ($u(t)=d_2(t)-d_1(t)$) and substituting on the Eq. 46, 47, 48 and 49 one gets, after collecting similar terms:

$$\frac{dd_1(t)}{dt}=\frac{k\left(\overline{T}-T_m\right)}{r\,l\,(1-f)\,u(t)}+\frac{q(t)}{r\,l} \qquad (50)$$

$$\frac{d\overline{T}}{dt}=\frac{2\left(T_r-\overline{T}\right)k}{r\,c_p\,f\,u(t)^2}+\frac{k\left(\overline{T}-T_m\right)\left(\left(\frac{\left(\overline{T}-T_m\right)}{l}-\frac{1}{c_p}\right)+\frac{q(t)}{r\,l\,u(t)}\right)}{r\,c_p\,(1-f)\,u(t)} \qquad (51)$$

$$\frac{dd_2(t)}{dt}=\frac{k}{r\,c_p\,f\,u(t)} \qquad (52)$$

$$\frac{du(t)}{dt}=\frac{1}{u(t)}\left(\frac{k}{r\,c_p\,f}+\frac{k\left(T_m-\overline{T}\right)}{(1-f)\,r\,l}\right)-\frac{q(t)}{r\,l} \qquad (53)$$

From this point on, there are two ways to solve the problem:

1) Numerical approach: to solve the set of Equations (Eq. 50 – 53) by any numerical method.

2) Analytical approach: to consider $\overline{T}$ as a constant value, calculated from Eq. 36 with $t=t_m$.

For the analytical approach, Eq. (51) is not used and Eq. (53) has the following solution for a constant heat flux:

$$u(t)=\frac{k}{q}\left(\frac{l}{c_p\,f}+\frac{\left(T_m-\overline{T}\right)}{(1-f)}\right)\bullet\left\{LambertW\left[\left(\frac{q}{k}\frac{u(t_m)}{\left(\frac{l}{c_p\,f}+\frac{\left(T_m-\overline{T}\right)}{(1-f)}\right)}-1\right)\exp\left(\frac{q}{k}\frac{u(t_m)}{\left(\frac{l}{c_p\,f}+\frac{\left(T_m-\overline{T}\right)}{(1-f)}\right)}-1\right)\exp\left(-\frac{q^2}{k\,r\,l}\frac{(t-t_m)}{\left(\frac{l}{c_p\,f}+\frac{\left(T_m-\overline{T}\right)}{(1-f)}\right)}\right)\right]+1\right\} \qquad (54)$$

Substituting this expression in Eq. 52 and solving it with $d_2(t_m)=u(t_m)$ as the initial condition one gets:

$$d_2(t)=u(t_m)+\frac{k\,l}{q\,c_p\,f}\ln\left(\frac{q}{k}\frac{u(t_m)}{\left(\frac{l}{c_p\,f}+\frac{\left(T_m-\overline{T}\right)}{(1-f)}\right)}-1\right)-\frac{k\,l}{q\,c_p\,f}\ln\left\{LambertW\left[\left(\frac{q}{k}\frac{u(t_m)}{\left(\frac{l}{c_p\,f}+\frac{\left(T_m-\overline{T}\right)}{(1-f)}\right)}-1\right)\exp\left(\frac{q}{k}\frac{u(t_m)}{\left(\frac{l}{c_p\,f}+\frac{\left(T_m-\overline{T}\right)}{(1-f)}\right)}-1\right)\exp\left(-\frac{q^2}{k\,r\,l}\frac{(t-t_m)}{\left(\frac{l}{c_p\,f}+\frac{\left(T_m-\overline{T}\right)}{(1-f)}\right)}\right)\right]\right\} \qquad (55)$$

Substituting Eq. 54 and 55 at the definition of $u(t)$ and rearranging the terms one gets:

$$
\begin{aligned}
d_1(t) = u(t_m) + \frac{k\,l}{q c_p f}\ln\left(\frac{q}{k}\frac{u(t_m)}{\left(\frac{l}{c_p f}+\frac{(T_m-\bar{T})}{(1-f)}\right)}-1\right) \\
-\frac{k\,l}{q c_p f}\ln\left\{LambertW\left[\left(\frac{q}{k}\frac{u(t_m)}{\left(\frac{l}{c_p f}+\frac{(T_m-\bar{T})}{(1-f)}\right)}-1\right)\exp\left(\frac{q}{k}\frac{u(t_m)}{\left(\frac{l}{c_p f}+\frac{(T_m-\bar{T})}{(1-f)}\right)}-1\right)\exp\left(-\frac{q^2}{k\,r\,l}\frac{(t-t_m)}{\left(\frac{l}{c_p f}+\frac{(T_m-\bar{T})}{(1-f)}\right)}\right)\right]\right\} \\
-\frac{k}{q}\left(\frac{l}{c_p f}+\frac{(T_m-\bar{T})}{(1-f)}\right)\left\{LambertW\left[\left(\frac{q}{k}\frac{u(t_m)}{\left(\frac{l}{c_p f}+\frac{(T_m-\bar{T})}{(1-f)}\right)}-1\right)\exp\left(\frac{q}{k}\frac{u(t_m)}{\left(\frac{l}{c_p f}+\frac{(T_m-\bar{T})}{(1-f)}\right)}-1\right)\exp\left(-\frac{q^2}{k\,r\,l}\frac{(t-t_m)}{\left(\frac{l}{c_p f}+\frac{(T_m-\bar{T})}{(1-f)}\right)}\right)\right]+1\right\}
\end{aligned}
\tag{56}
$$

Results

The electrical analogy analytical method developed in this paper was used to solve the ablation problem proposed by Landau. The two approaches for the solution of the problem, as described before, are used in this comparison. The ablating material considered is Teflon, which properties, the same used by Blackwell[9] in his work, are given in Table 2.

For both analytical approaches proposed, the main concern was the determination of the value of the parameter *f*. This parameter is very important because it determines the position of $\bar{T}$, which corresponds to the mean temperature of the solid as can be seen in Fig. 1 and the node 2 temperature (Fig. 3 and 4).

Fig. 5 shows a comparison between the present model and the numerical Blackwell[9] results for the temperature against position in a Teflon ablating material for several time instants and for several values of the parameter *f*. From this figure it is possible to observe that the theoretical curves using *f=0.6* compares better with the results of Blackwell[9]. The value of f (see Fig.1) was selected so that the beginning of the ablation was coincident for both analytical and literature numerical models. In Fig. 6, only Blackwell[9] results and the theoretical curve for *f=0.6* are presented. Both curves have basically the same slope, indicating that the process of ablation is well captured by the model proposed. In this plot, it is also possible to verify the advancing of the burning front with time, which is faster in the beginning of the ablation, quickly decreasing its velocity, that reaches a constant value.

Figure 7 shows similar to Fig. 5 curves, but for the analytical model with the use of the numerical approach for the solution of the system of equations. This equation system was numerically solved through an algebra computer software. In this case, the value of *f=0.75* was found to present the best comparison with the Blackwell[9] benchmark results. For comparison purposes, Fig. 8 shows only two curves: Blackwell[9] numerical and the mathematical results for *f=0.75*. This curve is similar to the one presented in Fig. 6. Comparing Figs. 6 and 8, one can see that the comparison shown in Fig. 6 is better. Also, analyzing Figs. 5 and 7 together, one can note that, comparing both models developed in the present work, the numerical approach is much more sensitive to variations of the parameter f than the analytical approach.

Table 2 – Teflon Thermophysic Properties and Test Parameters

r	$120\, lb_m / ft^3$
k	$0.000036\, Btu/ft\,s\,R$
c_p	$0.3\, Btu/lb_m\,R$
l	$1000\, Btu/lb_m$
T_m	$1500\, R$
T_r	$536\, R$
q	$250\, Btu/ft^2 s$

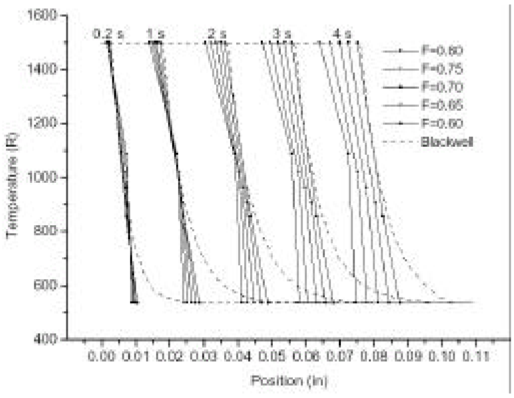

Fig. 5 - Comparison of the electrical analogy model using the analytical approach, for several f parameters, with Blackwell's results.

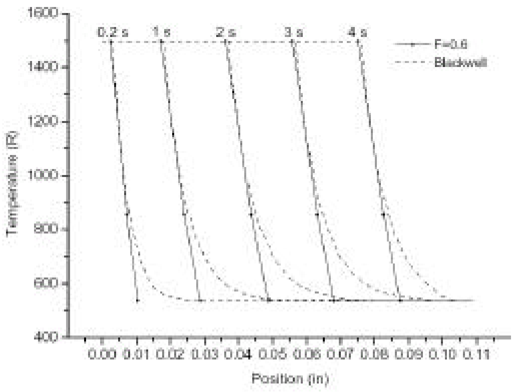

Fig. 6 - Comparison between the electrical analogy model, using the analytical approach and the best f parameter, and Blackwell's results.

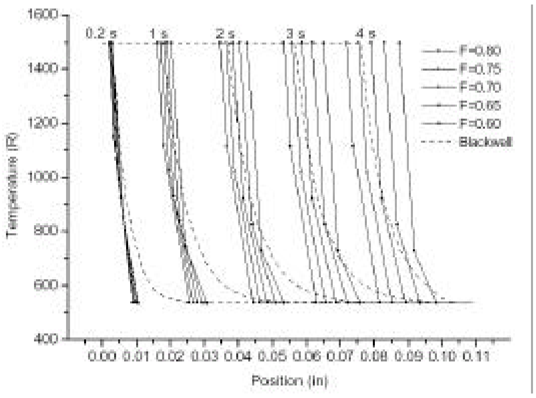

Fig. 7 - Comparison between the electrical analogy model, using the numerical approach with different values of the f parameter, and Blackwell's results.

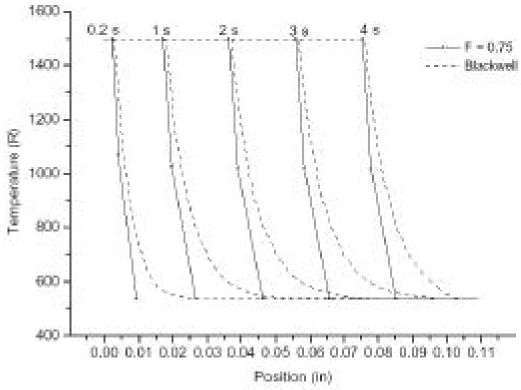

Fig. 8- Comparison of the electrical analogy method using the numerical approach and the best f parameter with Blackwell's results.

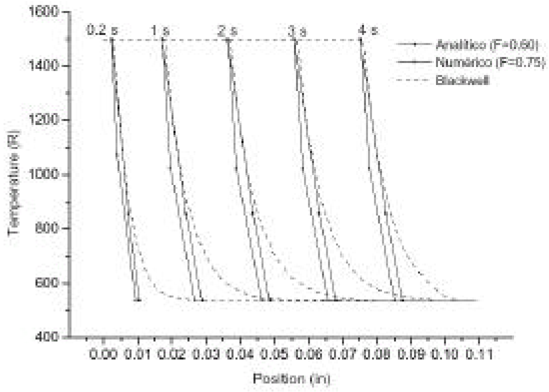

Fig. 9 - Comparison of the electrical analogy method using the analytical approach and numerical approach with the best f parameters and with Blackwell's results.

Figure 9 presents a comparison between the Blackwell results and the best curves obtained for the analytical and numerical approaches of the analytical model. One can note that the analytical procedure results have a better agreement with the benchmark data than the numerical approach. Both models can predict very well the ablation front, but the analytical approach presents a better comparison for the heat penetration front. The temperature difference that is found between the model proposed and the literature results can be attributed to the differences in the boundary conditions used at this front. In the case of the Blackwell model, the boundary condition considered was of the Newman type (insulation) far from the heat penetration front, while in the present model, the boundary condition considered was of the Newman type too, but exactly on the heat penetration front.

Conclusions

The electric circuit analogy method developed in the present work for one dimensional ablation problem showed to be a powerful tool for the determination of the burning front. Using the parameters described in Table 1, simple circuits can be constructed, even for more complex problems, and simple calculations fast can be performed, even in small computers. The lumped temperature of the nodes can be determined using the parameter $f=0.6$, for the analytical procedure, which presented the best results from the two approaches adopted for the temperature determination.

One should note that the traditional fully numerical computation can take long computational time to perform the same calculation. The comparison between the prediction of the heat penetration depth by the present model and the literature results is not good, due to the different boundary conditions adopted. The shape of the temperature curve as a function of the position is not an important parameter for the design of the reentry satellites protection systems, if sharp profiles are expected, as shown in this work. It is very important to note that, for the final validation of this method as well of any result shown in the literature, experimental results are necessary, especially to verify the boundary conditions adopted.

Acknowledgements

The authors wish to acknowledge CAPES for the financial and scholarship support (project PROCAD No. 0130/01-0), CNPq for providing the student scholarship and Brazilian Space Agency (AEB) for the financial support of this work.

References

[1] Chung, B.T.F., Hsiao, J.S., “Heat Transfer with Ablation in a Finite Slab Subjected to Time-Variant Heat Fluxes”, AIAA Journal, Vol. 23, Jan. 1985, pp. 145-150.

[2] Zien, T. F., “Integral Solutions of Ablation Problems with Time-Dependent Heat Flux”, AIAA Journal, Vol. 16, Dec. 1978, pp. 1287-1295.

[3] Horvay, G., “Freezing of a Growing Liquid Column”, Transactions of the ASME – Journal of Heat Transfer, Feb. 1960, pp. 37-47.

[4] Landau, H.G., “Heat Conduction in a Melting Solid”, Quarterly of Applied Mathematics, Vol. 8, 1950, pp. 81-94.

[5] Sunderland, J.E., Grosh, R.J., “Transient Temperature in a Melting Solid”, Transactions of the ASME – Journal of Heat Transfer, Nov. 1961, pp. 409-414.

[6] Goodman, T.R., “Application of Integral Methods to Transient Nonlinear Heat Transfer”, Advances in Heat Transfer, Vol. 1, Academic Press, New York, 1964, pp. 51-122.

[7] Biot, M.A., Agrawal, H.C., “Variational Analysis of Ablation for Variable Properties”, Transactions of the ASME – Journal of Heat Transfer, Aug. 1964, pp. 437-442.

[8] Storti, M., “Numerical modeling of ablation phenomena as two-phase Stefan problems”, International Journal of Heat and Mass Transfer, Vol. 38, No. 15, 1995, pp. 2843-2854.

[9] Blackwell, B.F., “Numerical Prediction of One-Dimensional Ablation Using a Finite Control Volume Procedure with Exponential Differencing”, Numerical Heat Transfer, Vol. 14, 1988, pp. 17-34.