EuroSun2004 - The 5th ISES Europe Solar Conference
International Solar Energy Society
20 – 23 June, Freiburg, Germany

Uncertainty calculation applied to different regression methods in the quasi-dynamic collector test

Manfred Georg Kratzenberg[1], Hans Georg Beyer[2], Sergio Colle[1], Dirk Petzoldt[1,2]
[1]Solar Energy Laboratory Federal University of Santa Catarina, Department of Mechanical Engineering, Florianópolis, Brazil
[2] Department of Electrical Engineering, University of Applied Science Magdeburg-Stendal, Magdeburg, Germany

1. INTRODUCTION

Testing the efficiency of solar collectors is a basic pre-requisite to obtain performance characteristics for thermal solar collectors.
In order to quantify those characteristics, we must run a test to determine the efficiency curve coefficients of those solar collectors.
In an international context, standards for the respective test procedures are given by EN 12975 [1] and ISO 9806 [2]. In Brazil the standard is given by the NBR 10184. Only the EN 12975 provides different test procedures, the Steady-State Test (SST) and the Quasi-Dynamic Test (QDT). The QDT has the advantage of allowing the execution of more collector tests within the same time period, with the same test equipment and the same test facility as compared to the steady state collector test [9]. On the other hand the quasi-dynamic test requires a somewhat more demanding effort for the calculation of the collector coefficients.
There has already been made a comparison between SST and QDT [7], but there have been no statements about the uncertainties. The purpose of the present work is to discuss and evaluate of test procedures in view of the uncertainty of the determination of the collector coefficients for the QDT. The *Least Square (LS)* and the *Weighted Least Square (WLS)* regression methods applied in the trend setting QDT are presented in this paper. Uncertainties of the *LS* and *WLS* results are calculated. A methodology for the verification of real confidence limit of the *LS* and *WLS* uncertainties is presented.

2. REGRESSION METHODS FOR THE DETERMINATION OF COLLECTOR COEFICIENTS

The Euronorm [1] proposes the *Multilinear Regression (MLR)* for the **QDT**. The efficiency of the collector is determined by a model containing 6 parameters (1). The six variables of the model have to be determined from the measured data. With the regression, the six collector coefficients can be determined. By minimizing the square errors of the differences between the calculated and the modeled efficiencies of all data pairs the coefficients are determined. As the uncertainties of the primary measurement data (radiation, flow rate, temperatures) join the calculation of the efficiency, a procedure would be useful, which considers

appropriate weighting of a data pair in respect to the inherent uncertainty of that data pair.
For example, pyranometers with an offset of approximately 10 W/m² show a higher relative measuring uncertainty in their lower measurement range than in their upper range. The *weighted least square method (WLS)* weights "measuring points by lower radiation (<300 W/m²)" less than for higher radiation for the regression process. Articles [5], [6] and [8] show that the *MLR regression* based on the *least square method (LS)* is not sufficient enough to determine the collector coefficients with their uncertainties for the **SST**. This paper shows the advantage of *WLS* for the *QDT*. In the following we discuss the different results obtained by the *LS* and the *WLS* for the **QDT**.

3. THE WEIGHTED LEAST SQUARE METHOD USED ON THE QDT

WLS is a weighted regression method, as the name says. The method is used for collector coefficients and uncertainties calculations. Theoretical background of this method is given by Press [3]. Compared to *LS* it has the following advantages:

- the collector coefficients are determined also using the measurement uncertainties of the transducers
- the measurement uncertainty of the transducer can be different in different measurement ranges
- the statistic distribution of the measurement uncertainties doesn't have to be normally distributed
- measurement uncertainties may be weakly correlated with each other

Equation (2) shows the individual error squares as function of the collector coefficients **a1...a6** and the measurement variables $\mathbf{X_1 .... X_6}$. Like in the *least square method (LS)*, first this „squares of the deviation between model and measurement" (2) have to be determined also by the *WLS*. To do this, the variables for the collector model have to be calculated with the data from the measurement (1). The initial coefficients could be determined by the *LS* regression.
In the next step, the uncertainties of all determined **error**–values $\mathbf{U_{error}[i]}$ have to be calculated for every 5 min mean value (3). The measurement uncertainty of every variable $\mathbf{U_{\eta me}; U_{x1} ... U_{x6}}$ has to be determined with help of the measurement uncertainties of the transducers to calculate the combined uncertainty $\mathbf{U_{error}}$ (3). After the squared quotients of the **error-** and $\mathbf{U_{error}}$**-values** have been determined, the new weighted evaluation function can be implemented (4).
The collector coefficients are varied iteratively by a spreadsheet program (e.g. EXCEL™) in a way that the sum of χ^2 is minimized. If the uncertainty of the measurements $\mathbf{U_{error}}$ is very large for one 5 min mean value, the weight or participation of this data point in the regression process is reduced due to the decreased contribution of the respective error value to the overall value of χ^2 (see equation 4). That works because the χ^2-value of this data point is lower.

$$\eta_c = \overbrace{\underbrace{\frac{\eta_0^{\,*}\cdot G_b}{G}}_{\text{beam model}} + \underbrace{\frac{\eta_0^{\,*}\cdot b_0\cdot}{G}\cdot\overbrace{\left(\frac{1}{\cos\theta}-1\right)}^{IAG_b}G_b}_{\text{angle of beam model}} + \underbrace{\frac{\eta_0^{\,*}\cdot IAM_{Gd}\cdot G_d}{G}}_{\text{diffuse model}}}^{\text{optical properties}} + \overbrace{k_1\cdot\frac{\Delta T}{G} + k_2\frac{(\Delta T)^2}{G}}^{\text{heat loss properties}} + \overbrace{\frac{k_3}{G}\cdot\frac{dT_m}{dt}}^{\text{thermal capacity p.}} \quad \textbf{(1)}$$

<u>Variables 1 - 7:</u>

1: $G_b / G = X_{1[i]}$; 2: $IA_{G_b} / G = X_{2[i]}$; 3: $G_d / G = X_{3[i]}$; 6: $\Delta T/G = X_{4[i]}$;

5: $\Delta T^2/G = X_{5[i]}$; 6: $\partial T_m / (\partial t \cdot G) = X_{6[i]}$; 7: $\eta_{me[i]} = \frac{\overbrace{\dot{Q}_m}^{\text{collector power}}}{\underbrace{G\cdot A}_{\text{solar power}}} = \frac{\dot{m}\cdot C_p \cdot (T_{out} - T_{in})}{G\cdot A}$

$\eta_{me[i]}$: measured efficiency; $\eta_{mo[i]} = \eta_{c[i]}$: modeled or calculated efficiency;
G_d : beam radiation $[W/m^2]$; G_b : diffuse radiation $[W/m^2]$; $G_b = G - G_d$
T_m : average collector temperature $= T_m = (T_{in} + T_{out}) / 2$ [°C]; $\Delta T = T_m - T_a$
T_a : ambient temperature; θ : incidence angle

<u>Coeficients $a_1 ...... a_6$ obtained from the regression</u>:

1: $\eta_0^{\,*} = a_1$: zero loss efficiency [-],
2: $\eta_0^{\,*} \cdot b_0 = a_2$: (b_0 : factor to determine the incident angle modifier of the beam irradiance [-]),
3: $\eta_0^{\,*} \cdot IAM_{Gd} = a_3$: (IAM_{Gd} : incident angle modifier for diffuse radiation [-])
4: $k_1 = a_4$: heat loss coeficient [W/(m^{2}K)], negative
5: $k_2 = a_5$: heat loss coeficient [W/(m^{2}K)], negative
6: $k_3 = a_6$: coeficient for the thermal capacity [kJ/(m^{2}K)]

$$(erro[i])^2 = \left(\eta_{me[i]} - \eta_{mo[i]}\right)^2 = \left(\eta_{me[i]} - \sum_{k=1}^{6}\left(X_{k[i]}\cdot a_k\right)^2\right)^2 \quad \textbf{(2)}$$

$$\left(u_{erro}[i]\right)^2 = \left(\frac{\partial(erro[i])}{\partial(\eta_m[i])}\cdot U_{\eta_m}[i]\right)^2 + \sum_{k=1}^{6}\left(\frac{\partial(erro[i])}{\partial X_{1[i]}}\cdot U_{xk}[i]\right)^2 \quad \textbf{(3)}$$

$U_{X1[i,1]}$........ $U_{X6[i,6]}$ = uncertainty variable 1 - 6 ; $U_{\eta me}[i]$ = uncertainty variable 7

$$\chi^2 = \sum_{i=1}^{N}\frac{(erro[i])^2}{(U_{erro}[i])^2} \quad \textbf{(4)}$$

N : quantity of 5 min mean values

Calculation of the collector coefficients uncertainties with WLS-method:

For the determination of the uncertainties of collector coefficients in the framework *WLS method* the method described by [3] is applied here. The application of this method for the **SST** is described by article [6]; here it is used for the **QDT** and calculated the uncertainties for the collector coefficients that were determined by the *WLS regression*. With equation (5) the collector coefficients uncertainties are calculated using the variables $X_{k(i)}$ from a data set of one collector test and the uncertainties $U_{error}[i]$ (3) of each "5 min mean value" from the same data set. The author [5] shows that $C = (A^T \cdot A)^{-1}$ of eqn. (5) yields a 6x6 matrix with squared uncertainties of the free parameters as diagonal elements and the co-variances between the parameters as off-diagonal elements.

$$A = \begin{pmatrix} \left\langle \frac{X_{1[1]}}{u_{erro[1]}} \right\rangle & \left\langle \frac{X_{2[1]}}{u_{erro[1]}} \right\rangle & \langle \ldots\ldots \rangle & \left\langle \frac{X_{6[1]}}{u_{erro[1]}} \right\rangle \\ \left\langle \frac{X_{1[2]}}{u_{erro[2]}} \right\rangle & \left\langle \frac{X_{2[2]}}{u_{erro[2]}} \right\rangle & \langle \ldots\ldots \rangle & \left\langle \frac{X_{6[2]}}{u_{erro[2]}} \right\rangle \\ \langle \ldots\ldots \rangle & \langle \ldots\ldots \rangle & \langle \ldots\ldots \rangle & \langle \ldots\ldots \rangle \\ \left\langle \frac{X_{1[n]}}{u_{erro[n]}} \right\rangle & \left\langle \frac{X_{2[n]}}{u_{erro[n]}} \right\rangle & \langle \ldots\ldots \rangle & \left\langle \frac{X_{6[n]}}{u_{erro[n]}} \right\rangle \end{pmatrix} \qquad \textbf{(5)}$$

$n = \text{number of measurement points}$

4. VERIFICATION OF THE CONFIDENCE LIMIT

Assume the case that the measured efficiency η_{me} is equal to the modeled efficiency η_{mo} during the whole test, we will get a straight line as presented in Figure 1. This line we call identity line. In the real case (Figure 2 and Figure 3) η_{me} and η_{mo} are different**.** Measured and modeled efficiency can be determined with their uncertainties for a chosen confidence limit. A confidence limit of 95% is chosen here. Combining the uncertainties of η_{me} and η_{mo} we get an uncertainty ellipse for each measuring point. The uncertainty ellipse shows, with a probability of 95%, where the single measuring point can be located. If the uncertainty ellipse has an intersection with the identity line, we assume for this measuring point that the measured efficiency is equal to the modeled efficiency. To prove the value and the confidence limit of the estimated uncertainties, we tested for which percentage of the pairs (measured / modeled efficiency) the uncertainty ellipse (with 95% confidence limit) covers the identity line with:
$\eta_{measured} = \eta_{modeled}$ (see figure 1).

The intersections of the ellipse were calculated using equation (6).

$$\eta_{MeSP}[i] = \pm\sqrt{\left(1-\frac{(\eta_{MeSP}[i]-\eta_{Me}[i])^2}{U_{\eta Me}^2[i]}\right)U_{\eta Mo}^2[i]} + \eta_{Mo}[i]$$

$\eta_{MeSP}[i]$: intersection of the identity line with the elipse **(6)**

$\eta_{Me}[i];\ \eta_{Mo}[i]$: pair of the modeled and measured eficieny

$U_{\eta Me[i]}\ ; U_{\eta Mo}[i]$: measured and modeled uncertainties of the eficiencies

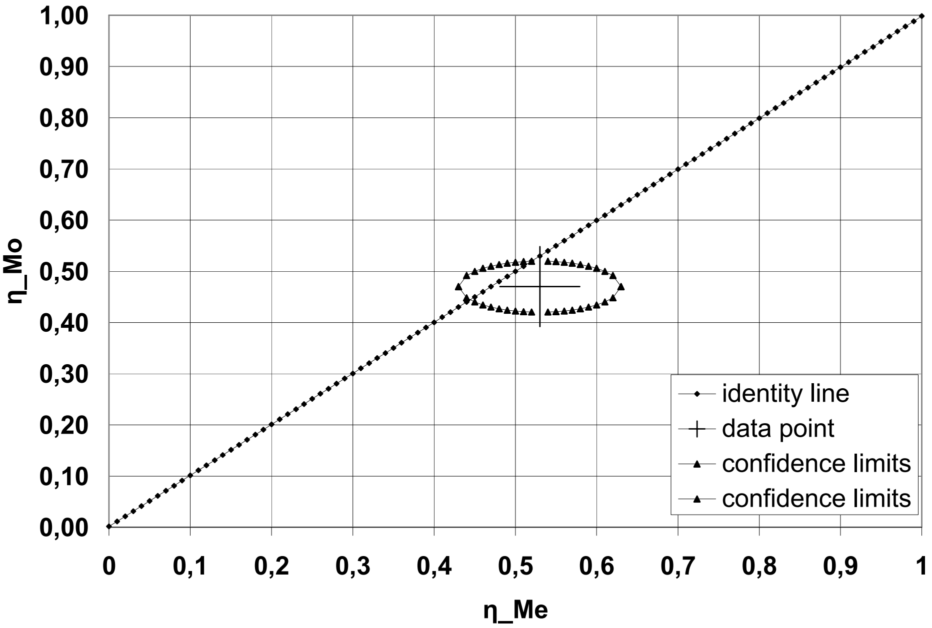

Figure 1: Example of one data point with his uncertainties within a 95% confidence limit

The modeled uncertainties for the i-th data point were calculated with:

$$\left(u_{\eta Mo}[i]\right)^2 = \sum_{k=1}^{6}\left(\frac{\partial(\eta Mo[\mathrm{i}])}{\partial X_{k[i]}}\cdot U_{\mathrm{xk}}[i]\right)^2 + \sum_{k=1}^{6}\left(\frac{\partial(\eta Mo[\mathrm{i}])}{\partial a_{k[i]}}\cdot U_{\mathrm{ak}}[i]\right)^2$$

$\mathrm{U}_{\mathrm{X1[i]}}$........ $\mathrm{U}_{\mathrm{X6[i]}}$: uncertainity of the variables; **(7)**

$\mathrm{U}_{\mathrm{X1[i]}}$........ $\mathrm{U}_{\mathrm{X6[i]}}$: uncertainity of the coeficients

The uncertainty of the coefficients is taken from the vector C as discussed above for the *WLS method*. For the *LS method* we take the uncertainty of the coefficients from the result of the Excel[TM] spread sheet program.
We calculated the measured uncertainty for the i-th data point with equation (8).

$$\left(u_{\eta Me}[i]\right)^2 = \left(\frac{\partial(\eta Me[\mathrm{i}])}{\partial \dot{Q}_{m[i]}}\cdot U_{\dot{Q}m}[i]\right)^2 + \left(\frac{\partial(\eta Me[\mathrm{i}])}{\partial G_{[i]}}\cdot U_{\mathrm{G}}[i]\right)^2 + \left(\frac{\partial(\eta Me[\mathrm{i}])}{\partial A_{[i]}}\cdot U_{A}[i]\right)^2$$

$U_{\dot{Q}m}[i]$: Uncertainty of the collector power **(8)**

$U_{\mathrm{G}}[i]$: Uncertainty of the global radiation

$U_{A}[i]$: Uncertainty of the collector area

Standard transducer uncertainties as defined by EN 12975 [1] are used for the calculation of equations (7) and (8).
For about 4,5% with the *LS method* and for about 7% with the *WLS method* the uncertainty ellipse does not cover the identity line (expected value is 5%). In the examples discussed here, we tested 134 of the efficiency pairs from one collector test. To get the 95% confidence limit result with both methods, we had to increase the uncertainty of the pyranometer from 10 W/m² to 10 W/m² + 1%. This indicates that the estimated uncertainty assumes the same value, with the same confidence limit (95%), with which the uncertainty was calculated using the *WLS* and the *LS methods*. The outliner points for the *LS* and the *WLS methods* are indicated in Figure 2 and Figure 3.

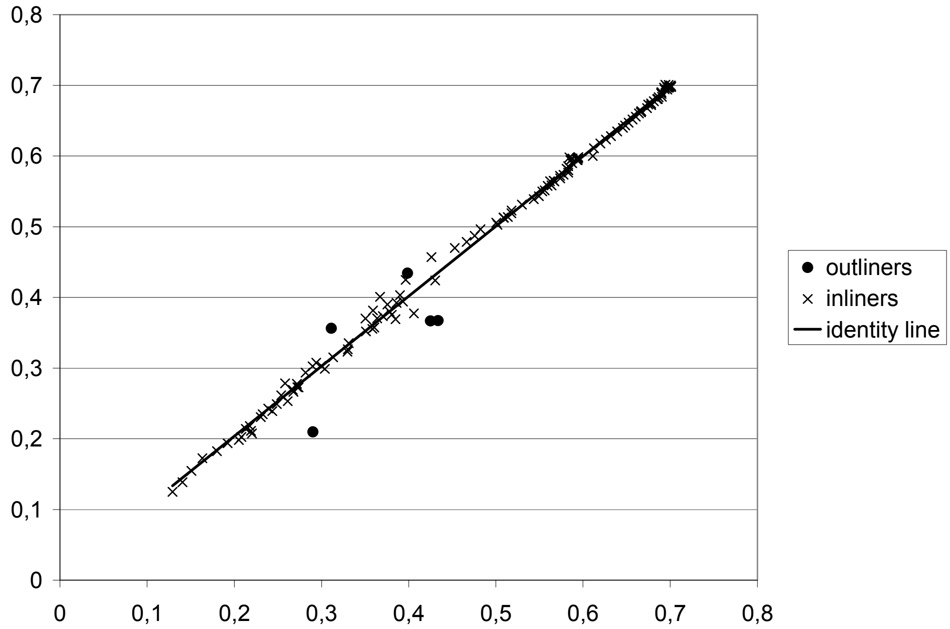

Figure 2: LS data points and outliners

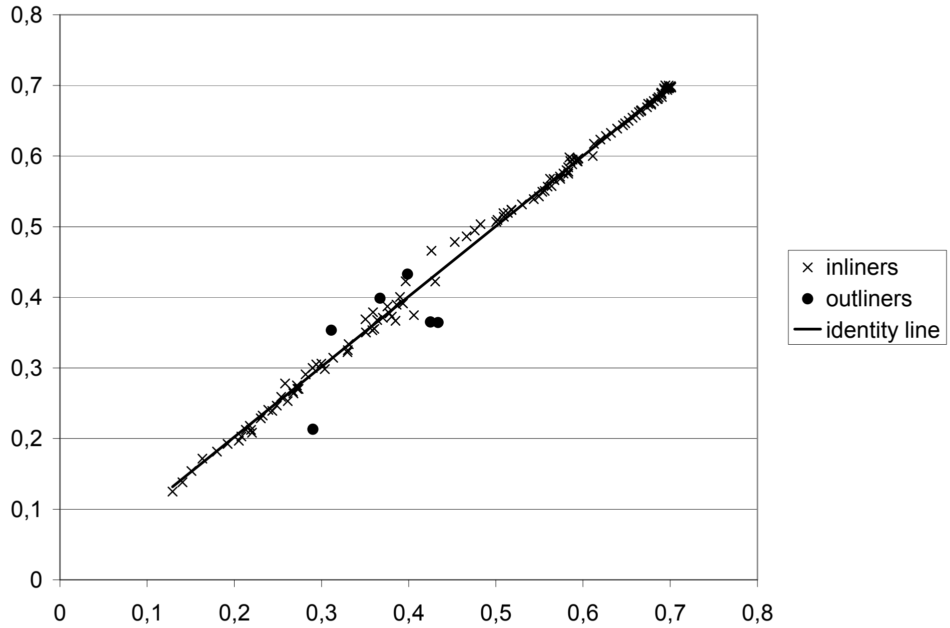

Figure 3: WLS data points and outliners

5. NORMALIZED EFFICIENCY CURVE

With equation (9) and (10) we can calculate the normalized efficiency curve. EN 12975 [1] defines the following conditions for that curve:

- beam radiation: 680 W/m² (85% of the global radiation)
- diffuse radiation: 120 W/m² (15% of the global radiation)
- global radiation: 800 W/m²
- Incidence angle: 15°

$$\eta_{0\_norm} = \eta_0 \cdot \left(\frac{G_b}{G} \cdot IAM_{dir\_15} + \frac{G_d}{G} \cdot M_{Gd} \right) ; IAM_{dir\_\theta} = 1 - b_0 \cdot \left(\frac{1}{\cos\theta} - 1 \right) \tag{9}$$

$$\eta_{norm} = \eta_{0\_norm} - k_1 \cdot \frac{T_m - T_a}{G} - k_2 \cdot \frac{(T_m - T_a)^2}{G} \tag{10}$$

Table 1: Collector coefficients with uncertainties in the 95% confidence interval

	LS				WLS			
	min	coeff.	max	U	min	coeff.	max	U
η_0	0,710	0,716	0,722	0,006	0,707	0,713	0,718	0,005
b_0	0,119	0,144	0,170	0,026	0,106	0,128	0,149	0,022
I_{AMdfu}	0,827	0,868	0,908	0,041	0,856	0,894	0,933	0,038
k_1	-6,445	-5,890	-5,335	0,555	-6,532	-6,109	-5,686	0,423
k_2	-0,049	-0,038	-0,027	0,011	-0,043	-0,035	-0,027	0,008
C_{eff}	-3821,2	-636,0	2549,1	3185,1	-3039,4	-612,1	1815,2	2427,3

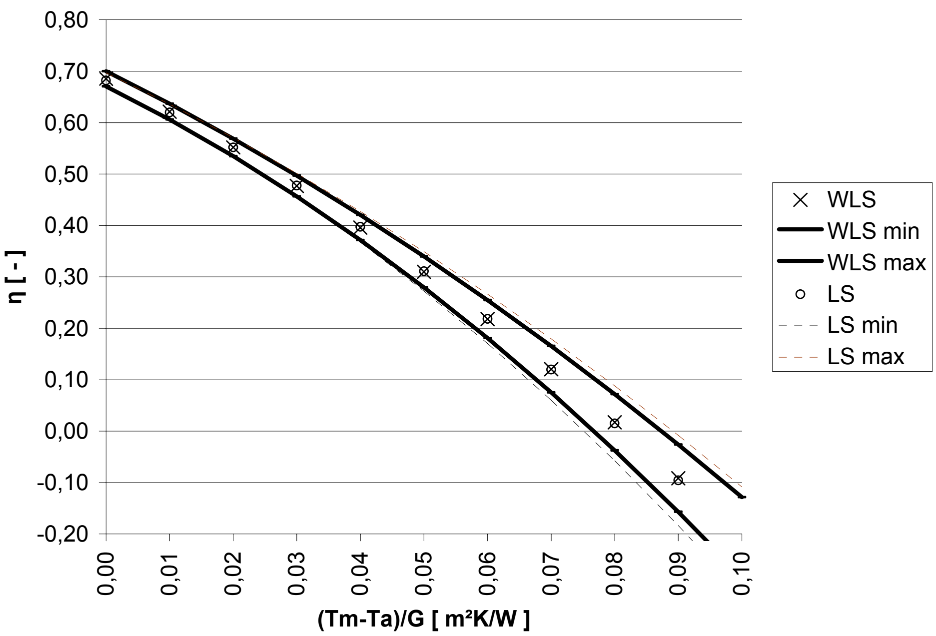

Figure 4: Normalized efficiency curve η with his uncertainty curves. η_{max} and η_{min} give the 95% confidence interval for the calculated efficiency.

The uncertainty of each point of the normalized efficiency curve is calculated with equation (11). This equation is comparable to equation (7). With the collector coefficients determined by the **quasi-dynamic test**, it is also possible to calculate the equivalent normalized efficiency curve of a **steady state test**.

$$\left(u_{\eta\, norm}[i]\right)^2 = \sum_{k=1}^{5} \left(\frac{\partial(\eta Mo[\mathrm{i}])}{\partial X_{k[i]}} \cdot U_{\mathrm{xk}}[i] \right)^2 + \sum_{k=1}^{5} \left(\frac{\partial(\eta Mo[\mathrm{i}])}{\partial a_{k[i]}} \cdot U_{\mathrm{ak}}[i] \right)^2 \quad \textbf{(11)}$$

6. DISCUSSION OF THE RESULTS

If we compare the measured individual efficiency points with those calculated using the coefficient sets from both the *LS* and the *WLS method*, we see that there are no big deviations (**errors)** throughout most of the test. This holds for both methods.

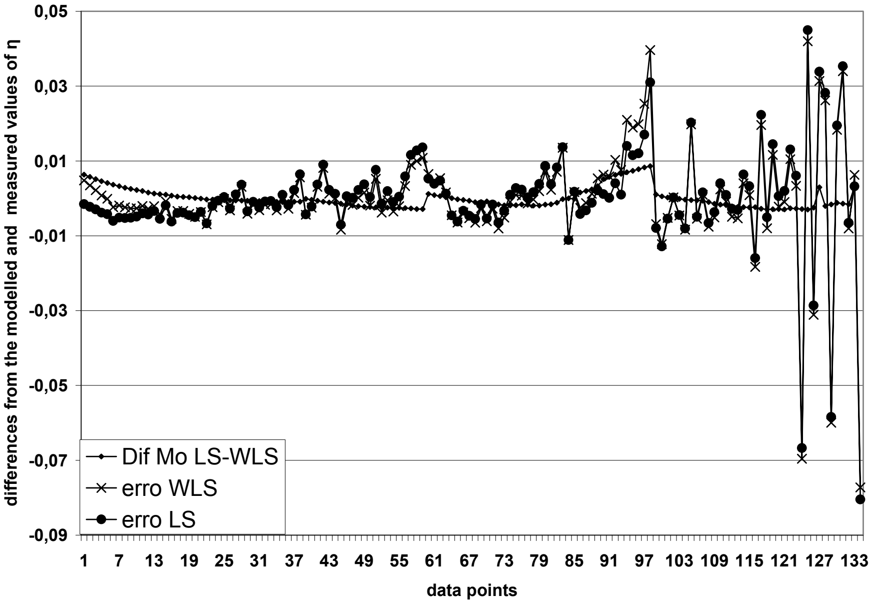

Figure 5: Differences of measured and modeled efficiencies using the coefficients resulting from both regression methods.

We emphasize that the data sets were selected by the reference conditions provided by EN 12975 [1] which are:

- solar radiation between 300 and 1100 W/m²
- wind speed 2 to 4 m/s
- T_{out} - T_{in} > 1 K
- dT_m/dt less than ±0,005 K/s
- only selection of values with positive power balance (positive η-values)
- stable mass flow of ±1% during a test day or test sequence
- stable mass flow of ±10% during the whole collector test
- stable input temperature of ±1 K in a valid data period
- $T_m = T_a \pm 3$ K for the data set from the clear sky day

The selection was extended by:

- the exclusion of values that can generate **errors** caused by heat flow direction changes (with positive η-values): only data with positive ΔT values are selected
- the exclusion of values where diffuse radiation fraction is > 50%
- the exclusion of the first and the last data-point (5 min mean value) of each of the valid data period, to ensure distance to the non-valid data points

Table 1 shows the coefficients, which were determined using *LS* and *WLS*. The uncertainties presented for the *LS* and the *WLS* methods in a confidence interval of 95% were calculated. The uncertainties and the highest and lowest possible coefficients provided by these uncertainties are also shown in the table.
The results of both methods are comparable. In figure 5 we show the error values (applying the two different regression methods) between the modeled and the measured efficiency for the whole test data set. The curve "Dif Mo LS-WLS" presents the differences between the results of the two models (*WLS* and *LS*).
As the first 90 measuring points (figure 5) where taken with nearly *clear sky conditions*, it is most likely that the collector's *optical behavior* (determined by the coefficients η_0, b_0 and M_{diff}) were determined reliable. This is also shown by the small uncertainty values of these parameters (table 1) and the small error values (figure 5).
In Figure 4 the results of the *LS* and *WLS methods* are given by a normalized efficiency curve. The normalized curve was calculated with equation (9) and (10). From table 1 we can see that the coefficient for the thermal capacity has a relatively high uncertainty. That uncertainty does not appear in the normalized efficiency curve (Figure 4). This because only the uncertainty of the non-dynamic coefficients $\mathbf{Ua_1 \ldots Ua_5}$ and variables $\mathbf{Ux_1 \ldots Ux_5}$ are used for the calculation of the total uncertainty with equation (11).

7. CONCLUSION

Figure 4 and Table 1 show that the *weighted least square fitting method* has lower uncertainties than the *least square fitting method*. This proves the hypothesis that the *WLS method* leads to better regression results. In our example, the optical coefficients of the *WLS fit* do not differ much from the coefficients obtained from the *LS fit*. The small differences of the normalized efficiency curve are mainly a result of the differences in the heat loss coefficients from *WLS* as compared to the *LS*. We have to remark that the present results are only based on one collector test. In further collector tests with other collectors the differences of the coefficients may be higher. Anyway the *WLS method* is the method that delivers more adequate results for the **quasi-dynamic test**. The post verification of the 95% confidence limit shows that the uncertainty calculations are correct, and gives more credit to the calculated uncertainty of the test results.

8. SUMMARY AND OUTLOOK

We calculate the collector coefficients and their uncertainties with the *least square* and the *weighed least square methods* for a **quasi-dynamic test** [1].
The 95% confidence limits for each collector coefficient are calculated as well as the respective limits for the collector efficiencies resulting from the identified model.
Using the test data, the confidence limits for the efficiencies can be validated, which proves that the uncertainties of the “collector coefficients“ as well as the “efficiency curve“ of the collector can be determined in a reliable way. However, a review of these uncertainty statements still requires a more comprehensive analysis of several complete tests.
The *weighted least square method (WLS)* gets slightly different coefficients with the same collector as the *least square method (LS)*. It should be the more accurate one, as mentioned earlier. This statement has yet to be supported by the use of a more extended database.
Like both the **quasi-dynamic** and the **steady-state tests** are performed under outdoor conditions, from the measurement conditions Point of view it is impossible to repeat them, because weather conditions (combination of solar radiation and ambient temperature) always vary. Thus, to get a quantitative statement on the test reproducibility, the need for an extended database is again underlined. It should be remarked that this reproducibility is strictly related to the defined standard test conditions (or data selection conditions) as given by EN 12975 [1].
The easy implementation in EXCEL™ spreadsheet makes it possible to apply the *WLS method* regularly for collector test evaluations.

Acknowledgements: The test data was gained using the equipment of ITW at the University of Stuttgart, for which we express our gratitude.

9. REFERENCES

[1] CEN (2000) Standart 12975-2: Solar thermal systems and components - Solar collectors - Part 2: Test methods, European Committee for Standardisation.

[2] ISO (1994) Standard 9806-1: Test Methods for Solar Collectors. Part 1: Thermal Performance of Liquid Heating Collectors Including Pressure Drop. ISO, Switzerland.

[3] Press W., Teukolsky S.A., Vetterling W.T., Flannery B.P. (1996). Numerical Recipes, 2nd ed. Cambridge University Press, Oxford.

[4] NBR 10184 (1988) Coletores solares planos para líquida - Determinação do rendimento térmico, ABNT, Brazil.

[5] Mathioulakis E., Voropoulos K., Belessiotis V. (1999) Assessment of uncertainty in solar collector modeling and testing, Solar Energy Vol. 66 No. 5, 337-347.

[6] Müller-Schöll Ch., Frei U. (2000) Uncertainty analyses in solar collector measurements, Proc. of the Eurosun 2000.

[7] Fischer S., Heidemann W., Müller-Steinhagen H., Peters B., Berquist A. (2001) Collector test method under quasi-dynamic conditions according to the European Standard EN 12975-2, Proc. ISES Solar World Congress.

[8] Sabatelli V., Marano D., Braccio G., Sharma V.K. (2002) Efficiency test of solar collectors: uncertainty in the estimation of regression parameters and sensitivity analyses, Energy Conversion & Management, Vol. 42.

[9] Kratzenberg M., Beyer H.G., Colle S. (2002) Setup of a test facility for the characterization of thermal collectors according to the Euronorm at the “Universidade Federal de Santa Catarina”, Proc. “Sun at the end of the world” International solar energy congress and exhibition, Universidad Técnica Federico Santa María.

[10] Kratzenberg M., Beyer H.G., Colle S., Albertazzi Gonçalves A. (2003) Test facility for quasi-dynamic collector tests for the characterization of thermal solar collectors in accordance with the international norms, metrologia-2003 – Metrologia para a Vida, Sociedade Brasileira de Metrologia (SBM).

[11] Kratzenberg M., Beyer H.G., Colle S., Petzoldt D. (2004) Bestimmung der Kollektorparameter und ihrer Unsicherheiten über die Methode der gewichteten Fehlerquadrate für den statischen und den quasi-dynamischen Kollektortest, Proc. Otti-Kolleg Thermische Solarenergie.