Analytic determination of the expanded uncertainty for steady state and quasi dynamic collector tests under outdoor conditions

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Abstract

In this paper the attempt to calculate the expanded uncertainties for the standardized outdoor collector tests is presented. With an extended series of collector tests and test data the confidences of the uncertainty of the collector model values are checked. With independent data the calculated prediction intervals and the uncertainty intervals of the mean response for each test are corrected with iteratively obtained correction factors leading to 95% confidence levels for both intervals. The expanded uncertainty budget for the QDT and SST tests is elaborated. The uncertainty budget is based on the variances, the expanded uncertainty of the random uncertainties of the transducers that measure the variables of the collector model and the measured thermal collector power as well as the systematic uncertainty of the transducers. Hereby the correlations of the regression coefficients are considered. For different collector operation temperatures the yearly expanded uncertainties of the SST and the QDT are compared and differences of the steady state and the quasi dynamic test are discussed.

Keywords: Outdoor collector tests, expanded uncertainties

1. Introduction

According to the Gleisdorf meeting [1], a generalized recommendation is given [2] to convert the thermal collector area of flat plate collectors into a figure for the installed thermal capacity. Using this calculation method to estimate the word wide installed capacity of thermal flat plate collectors, one can observe that e.g. for the year 2001 the power potential of the solar collectors is with 69320 MW even 3 times higher than potential of the wind power plants [3]. The worldwide installed collector are of 99 km^2 converts yearly 41795 GWh, that is equivalent to 18.16 Mt CO_2 reduction [4]. With a growth of 26 % (from 2000 to 2001) for the flat plate and evacuated tube collectors [4] and the recent technological transgress into the area of space heating and cooling using these devices [5], systems using fix mounted solar collectors during the operation represent one of the most important alternative renewable energy source.

To guaranty the return of the investments, the reliability and the efficiency of this important alternative renewable energy source has to be verified. The solar collector test [6, 7] is one of the most important issues to be accomplished in this framework. Solar collector tests are by the most institutions performed under outdoor conditions. According to ISO9806 these tests are performed under steady state conditions SST. This test requires high stability of the solar radiation and other ambient conditions as the ambient temperature and wind over the collector aperture. As a consequence, a solar collector test can take up until to three months, (that leads to the effect that the solar collector occupies the test rig during this time). Since 1997 the European standard EN12975 allows that test data under all weather conditions can be used for the evaluation of the collector efficiency test. Following the EN12975 this new test is called quasi dynamic collector test QDT. With this new option, a collector test can by performed within one week. But until today there are doubts about the uncertainty and compatibility of the results of the steady sate SST and the QDT collector tests. While for the steady state test the collector is characterized by a model using 3 coefficients, the underlying model for the quasi dynamic test needs 6 - 9 coefficients. The metrological procedures defined by the two standards are, however below the state of the art in metrology and statistics. Except for the uncertainties for the standard group of the transducers that are specified in the two standards, procedures for the calculation of the uncertainties of the results are not specified or recommended. Also, the standards are not specifying maximum uncertainties of the regression coefficients or the mean response of the collector model. As the quasi dynamic test is accomplished under variable weather conditions, one can expect that the QDT is connected to higher uncertainties than the SST. On the other hand, the test under quasi dynamic conditions accounts for a wide range of weather conditions and applies a more comprehensive collector model. Thus, it is expected that the collector performance under real operation conditions are better interpreted by the QDT results.

2. Model equations

Both standards [6] and [7] are based on the use of input data that represent mean values (taken over periods of 5 to 10) minutes for the QDT [6] and 15 min [6, 7] for the SST. The data of the same measurements are subject to test specific selection criteria, to obtain the useful data that are the input for the test evaluation (regression) using test specific collector models.

2.1 Collector model of the steady-state test

For the steady-state collector test model the power output of the collector is expressed by:

$$\underbrace{\dot{Q}_{\mathrm{mo}}}_{\hat{Y}=Y\ estimated} = \overbrace{\underbrace{\eta_0}_{a_1}\underbrace{G}_{X_1}}^{\text{optical property}} - \overbrace{\underbrace{k_1}_{a_2}\underbrace{\Delta T}_{X_2} - \underbrace{k_2}_{a_3}\underbrace{(\Delta T)^2}_{X_3}}^{\text{heat loss properties}} \tag{1}$$

Where $\dot{Q}_{\mathrm{mo}}$ [W/m^2] is the estimated collector power per collector aperture area *A*, $\dot{Q}_{\mathrm{mo}}$ is determined as the, by the model, estimated power value $\hat{Y}$ that is a function of the variables X_1, X_2 and X_3 with coefficients a_1, a_2 and a_3 of a linear model (see eqn. 1). The variables X_1, X_2 and X_3 are derived from measured quantities, that are the global radiation G [W/m^2] and the difference between the average collector temperature T_m and the ambient temperature T_a , denominated ΔT [K] (in this simplified model the average collector temperature T_m is calculated by $(T_{out}+T_{in})/2$). All measured quantities are taken from the experiment as mean values in 15 minutes intervals. With the regression variables X_1, X_2 and X_3 (eqn. 1) and the quantity $\dot{Q}_{\mathrm{me}}$ (eqn. 2) - all defined by the measured values - a linear regression problem to find the regression coefficients a_1, a_2 and a_3 is defined, where the regression coefficients (eqn. 1) are identified as the dimensionless zero loss coefficient η_0 and heat loss coefficients k_1 [W/m^{2}K] and k_2 [W/(m^{2}K^2)].

The coefficients have to be determined by a regression procedure as described below, using the deviations of estimated power $\dot{Q}_{\mathrm{mo}}$ and measured power $\dot{Q}_{\mathrm{me}}$ as criteria that are both taken per collector aperture area. In the regression procedure $\dot{Q}_{\mathrm{me}}$ is set as goal for the estimator $\hat{Y}$ or $\dot{Q}_{\mathrm{mo}}$ and is given by the following equation

$$\dot{Q}_{me} = \dot{m}\ C_p\ (T_{out} - T_{in})/A = \dot{V}\ \rho\ C_p\ (T_{out} - T_{in})/A \tag{2}$$

where T_{in} represents the inlet and T_{out} the outlet temperature, $\dot{m}$ is the mass flow, which is obtained by the multiplication of the volume flow $\dot{V}$ with the fluid density ρ (that is a function of the fluid temperature measured at the input of the volume flow transducer) and C_p is the heat capacity of the fluid (in our case water) that is a function of the mean fluid temperature T_m of the collector.

2.2 Quasi-dynamic collector model

To allow for the use of data taken under quasi-dynamic conditions the model and his parameters have to be modified accordingly to equation (3). Where a_1 to a_6 are the regression coefficients and X_1 to X_6 are the regression variables, all together used in the multiple linear regression of the quasi-dynamic collector test. G_d is the diffuse radiation, G is the global radiation, and G_b is the beam radiation calculated by the relation $G_b = G - G_d$, all measured in units of W/m^2.

$$\underbrace{\dot{Q}_{mo}}_{Y-estimated} = \overbrace{\underbrace{\underbrace{\overset{a1}{\eta_0}\,\overset{X1}{G_b}}_{\text{beam model}} + \underbrace{\overset{a2}{\eta_0\; b_0}\,\overset{X2}{\left(\frac{1}{\cos\theta}-1\right)G_b}}_{\text{correction for nonnormal incidence}}}_{\text{beam radiation model } = K_{\theta b}(\theta)\ \eta_0\ Gb} + \underbrace{\overset{a3}{\eta_0\; K_{\theta d}}\,\overset{X3}{G_d}}_{\text{diffuse radiation model}}}^{\text{optical properties}} - \overbrace{\underbrace{\overset{a4}{k_1}\,\overset{X4}{\Delta T} - \overset{a5}{k_2}\,\overset{X5}{\Delta T^2}}_{\text{heat loss properties}}^{\text{heat losses}} - \underbrace{\overset{a6}{k_3}\,\overset{X6}{\frac{dT_m}{d\tau}}}_{\text{thermal capacity p.}}}^{\text{thermal processes}} \quad (3)$$

The average collector temperature T_m, the difference between collector mean and ambient temperature ΔT, and the measured collector power per collector area $\dot{Q}_{me}$ are derived in the same way as in the steady-state collector model. In the accomplished QDT - test all mean values of the measurable quantities are taken from the measurements of the experiment in 5 minutes intervals. The coefficients to be determined by the regression are defined by: a_1 (η_0) that represents the dimensionless zero loss efficiency of the QDT model, a_2 that is obtained by the multiplication of η_0 with b_0, where b_0 is a collector specific factor, which scales the part of the beam radiation ($a^2 \cdot X^2$) that the collector is not able to convert in heating energy, because of the increasing absorption losses of the absorber as well as the reflection and transmission losses of the cover for incidence angles θ different to zero. The incidence angle θ is the angle between the normal position of the sun to the collector and the position for that the mean value of a sample is calculated. The coefficient a_3 is determined by the multiplication of η_0 with $K_{\theta d}$, which is the *mean incident angle modifier* considered for diffuse radiation, the regression coefficients a_4 (k_1) and a_5 (k_2) are the heat loss coefficients that using the same sub-models as already used in the SST model (eqn. 1) The regression coefficient a_6 (k_3) [J/m^{2}K] represents the overall heat capacity per collector area that accounts the heat capacity of the collector together with its fluid content.

3. Regression analysis

The set of the collector coefficients, obtained by the linear regression, is connected to uncertainties that have to be specified. The respective procedures to analyze the uncertainties of the estimates in a linear regression obtained by the classical least square method are e.g. given by ISO-GUM [8]. The regression technique, that is used to derive both, the regression coefficients and their uncertainties for linear models are shown in [8 to 11]. Based on [8] and [9 to 11] we can calculate the variances of the estimates of the model response for the linear model and use them for the uncertainty calculation of the thermal collector power [8]. Subsequently, the uncertainty of the respective energy production of the collector can be obtained.

3.1 Estimation of the regression coefficients

The basis for the regression procedure, used for the QDT and the SST collector test, is given by the equation for the *sum of square errors* SS_E (eqn. 4) of the modeled collector power $\dot{Q}_{mo}$ as compared to the measured power $\dot{Q}_{me}$, which has to be minimized,

$$SS_E = \sum_{i=1}^{n} (\in_i)^2 \rightarrow \min = \sum_{i=1}^{n} \left(\dot{Q}_{me,i} - \dot{Q}_{mo,i} \right)^2 = \sum_{i=1}^{n} \left(\dot{Q}_{me,i} - \sum_{j=1}^{k} \left(X_{j,i}\, a_j \right) \right)^2 \tag{4}$$

where $\in$ is the difference or error of the two power values. Here j = 1…k is the number of the model components in the multiple linear regression and i = 1…n counts the number of the used mean values (obtained from the samples of the experiment) within a regression. For the SST, k is 3, and for QDT, k is 6 for covered collectors. The regression coefficients a_1 to a_k are determined here by solving the linear regression model (eqn. 5).

$$\dot{Q}_{mo,i} = \hat{Y}_i = \sum_{j=1}^{k} a_j\, X_{i,j} \tag{5}$$

The numbers of the regression coefficients in the model are determined k. The used regression variables are $X_{i,j}$ (see also eqn.1 and eqn. 3). This equation can also be given as matrix expression:

$$\{\dot{Q}_{mo}\} = [X] \times \{a\} + \{\in\} = \begin{Bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{Bmatrix} = \begin{bmatrix} X_{11} & X_{12} & \cdots & X_{1k} \\ X_{X1} & X_{22} & \cdots & X_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ X_{n1} & X_{n2} & \cdots & X_{nk} \end{bmatrix} \times \begin{Bmatrix} a_1 \\ a_2 \\ \vdots \\ a_k \end{Bmatrix} + \begin{Bmatrix} \in_1 \\ \in_2 \\ \vdots \\ \in_k \end{Bmatrix} \tag{6}$$

3.2 Estimation of the uncertainty of the regression coefficients

The estimated *residual mean square error* (MSE) or $E(\sigma)^2$, which is the variance of the error term $\in$ of the regression is given by eqn. (7):

$$E(\sigma)^2 = \frac{\sum_{i=1}^{n} (\in_i)^2}{d_f} = \frac{\{\in\}' \times \{\in\}}{n-k} = \overbrace{\frac{\{\{Y\} - [X] \times \{a\}\}' \times \{\{Y\} - [X] \times \{a\}\}}{n-k}}^{\text{Matrix expression}} \tag{7}$$

where the residual degree of freedom is d_f is obtained by the subtraction of number of the regression variables k (that in for the presented regression equal to the regression coefficients) from the number of the total mean values that were selected form the measured data for the test evaluation (eqn. 7).

By using the mean square error and the data matrix *[X]*, according to references [9 to 11] the *variances of the estimators* for the coefficients are obtained as diagonal elements of the matrix given in eqn.(7). The matrix is formed by the regression variables X_{ij} of the whole collector test (see eqn. 5). The *diagonal elements* of the matrix refer to the variances $var(a_1)$ ... $var(a_k)$ and the *off diagonal elements* of these matrixes refer to the covariances $cov(a_1, a_2)...(a_1, a_k)$ of the estimators [9 to 11].

$$E(\sigma)^2 \times [[X'] \times [X]]^{-1} = \begin{bmatrix} \mathrm{var}(a_1) & \mathrm{cov}(a_1, a_2) & \cdots & \mathrm{cov}(a_1, a_k) \\ \mathrm{cov}(a_2, a_1) & \mathrm{var}(a_2) & \cdots & \mathrm{cov}(a_2, a_k) \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{cov}(a_k, a_1) & \mathrm{cov}(a_k, a_2) & \cdots & \mathrm{var}(a_k) \end{bmatrix} \quad (8)$$

where j = 1…k is the number of the used model components in the multiple linear regression**.** The standard error of the estimated regression coefficients $s_e(a_j)$ we thus obtain by the square root (see eqn. 8) of diagonal elements of this matrix (eqn.8).

$$s_e(a_j) = \sqrt{\mathrm{var}(a_j)} \quad (9)$$

Given the estimators of the coefficients $\hat{a}_j$, a 100(1-α) (i.e.95%) confidence interval for the regression coefficients a_j is determined by eqn.(9). Here $t_{\alpha/2,n-k}$ is the student value selected for the level of significance of α/2 and the degrees of freedom *n* - *k* (see also reference [9 to 11]). The regression uncertainties of a_j are defined by [8 to 11] with the following equation.

$$U_{regr}(a_j) = \pm\, t_{\alpha/2,n-k}\ s_e(a_j) = \pm\, t_{\alpha/2,n-k} \sqrt{\mathrm{var}(a_j)} = \pm\, t_{\alpha/2,n-k}\ u(a_j) \quad (10)$$

3.3 Estimation of the regression uncertainty of the mean response of the collector model

In different references concerned to statistic evaluations of the model response, for multiple linear regressions [8 to 11] one can find different evaluations with the validity for a certain confidence (e.g. 95%). The expanded uncertainty of the instantaneous power values, considering that the collector coefficients are mutually independent, is based on the variances of the instantaneous power values and can be obtained by two different calculations. The first is determined by ISO-GUM [8] and uses equation (11); ISO-GUM uses this method in a simple demonstration to obtain the expanded uncertainty of the correction curve in a temperature sensor calibration process and considers also the correlated uncertainty, originated by the correlation of the subcomponents from the linear model.

$$U_{regr}(\dot{Q}_{mo,i}) = \pm\, t_{\alpha/2,n-k}\, \sqrt{\mathrm{var}(\dot{Q}_{mo,i})} = t_{\alpha/2,n-k}\; u(\dot{Q}) = t_{\alpha/2,n-k} \sum_{j=1}^{k} \left(\frac{\partial(\eta_{\mathrm{mo,i}})}{\partial(a_k)} u_{\mathrm{ak}} \right)^2 \tag{11}$$

The second, based on the variance of the model response [9 to 11] is obtained by the expression (12) and is denominated confidence interval $CI(\dot{Q}_{mo,i})$.

$$U_{regr}(\dot{Q}_{mo,i}) = CI(\dot{Q}_{mo,i}) = \pm\, t_{\alpha/2,n-k}\, \sqrt{\mathrm{var}(\dot{Q}_{mo,i})} = \pm\, t_{\alpha/2,n-k}\, \sqrt{E(\sigma^2)\,\{X_0\}\left[[X]^T\,[X]\right]^{-1}\{X_0\}^T} \tag{12}$$

where the matrix [X] is the X-matrix of equation 6 and the vector $\{X_0\}$ is a row of that matrix, used to determine the uncertainty of the estimated power values that are modeled with the test data. If one analysis the equations (11) and (12), it is to see that both uncertainties are calculated multiplying the *student-t value* by the root of the variance of the mean response. The results obtained using the two different uncertainty estimation methods, are shown in Figure 1.

Apart from the confidence interval of the mean response of the multiple linear model, authors [9 to 11], determine the prediction interval given by equation 13.

$$PI(\dot{Q}_{mo,i}) = \pm t_{\alpha/2,n-k}\, \sqrt{\mathrm{var}(\dot{Q}_{mo,i} - \dot{Q}_{me,i})} = \pm t_{\alpha/2,n-k}\, \sqrt{E(\sigma^2)\,(1 + \{X_0\}\left[[X]^T\,[X]\right]^{-1}\{X_0\}^T)} \tag{13}$$

To estimate the uncertainty intervals of estimated power values, which are modeled with data that are independent to the collector test (e.g. TMY), the $\{X_0\}$- vector has to be build up with this independent data [9 to 11]. For the prediction interval the variance is calculated, based on the variance of the difference between the modeled and the measured value [11 and 12] of the mean response (equation 14).

$$\mathrm{var}(D_i) = \mathrm{var}(\dot{Q}_{mo,i} - \dot{Q}_{me,i}) = \mathrm{var}(\dot{Q}_{mo,i}) + E(\sigma^2) \tag{14}$$

4. Presentation of the results

In this section the results obtained by the application of the (equations 11 to 13) are presented in order to compare the estimations done by the different methods. Using the data of continuous operation and measurements with the same collector four quasi-dynamic tests and one steady state test, as well as 941 test independent data, that corresponds to QDT selection criteria, where selected.

The SST is only build up with four 15 min mean values for four different operation temperatures that leads to sixteen measured values within the complete test. Therefore for the statistic comparison of the two methods (equation 11 and 12), used for the estimation of the model response uncertainty, only data obtained from the QDT is used. Figure 1 shows an example for the uncertainty intervals occurring within a quasi dynamic test, calculated with the two different introduced methods, as well as the prediction interval and the residuals, obtained by subtraction sub-traction of the measured and the calculated collector power values.

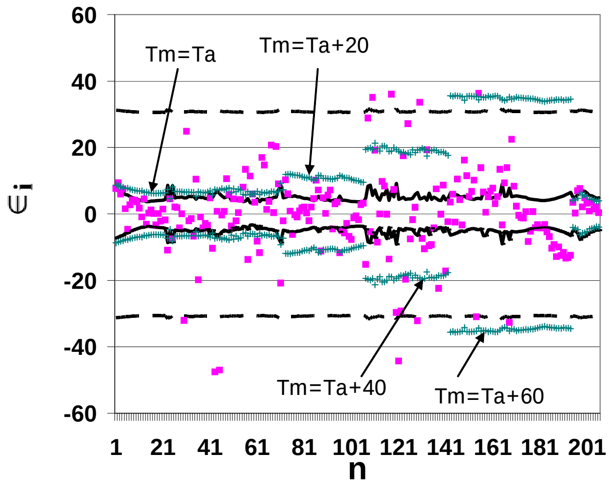

Figure 1: Errors $\in_i$ between the modeled and the measured thermal power values (residuals) together with its instantaneous uncertainty intervals and the prediction interval for 95 % of confidence

If the chosen confidence is estimated correctly, 95% of the differences i.e. the $\in_i$ - values have to be within the determined prediction intervals (equation 15).

$$-PI(\dot{Q}_{mo,i}) > \in_i > PI(\dot{Q}_{mo,i}); \qquad \in_i = \dot{Q}_{me,i} - \dot{Q}_{mo,i} \tag{15}$$

The prediction interval (outer interrupted line in figure 1) is useful to know the range within which the model predicts the power for the instantaneous values. It can be seen that approximately 5 % of the data points are outside of the interval. For the confidence interval of the mean response (inner continuous line) more than 5 % of the $\in_i$ – values are outside. With the confidence interval i.e. the uncertainty of the mean response, it is only possible to determine a prediction interval of the calculated energy (equation 15) using the collector test [12]. The uncertainty intervals that are distinguished in figure 1 by the different collector operation temperatures (T_m) are calculated with the ISO-GUM method (equation 11). It can be seen that for no heat losses (T_m = T_a), the uncertainty intervals calculated with equation (11) and (12) are comparable. By increasing heat losses the ISO-GUM method increase the uncertainty interval even over the calculated values that are defined for prediction interval (equation 13). Taking only the standard deviation of the $\in_i$ – values, there are for all operation temperatures closed to the confidence intervals, which indicates that the uncertainty intervals calculated with equation (10) are overestimated if one like to use them for the prediction estimations of the energy calculations (integration of the power values over the time). Therefore further calculations are only uses the method determined by the authors [9 to 11].

5. Confidence of the model

5.1 Real confidence

If the measurements, the regression method and the model are correctly selected and applied [6, 7], 95% of the residual differences $\in_i$ should be localized within the prediction interval (equation 13). This should also hold for test independent measured data. The initially assumed confidence of a collector test can in this way be checked by the application of the extracted collector model to the data of the proper collector test, or to independent data, not used in the actual test. As the model was not trained with this independent data, it is expected, that the model performs with lower confidence than with the proper test data.

5.2 Correlation effects

As pointed out in [7], [12] and [16], the linear sub-models of the regression model, e.g. for the heat loss properties are exposed to correlation effects that occur as the regression coefficients are always estimated in a complete set for each regression. Therefore the regression coefficients show correlation [9].

5.3 Adjusted confidence intervals

In [9 - *A6 to A8*] tables that are elaborated numerically are presented, which can be used to correct the confidence and the prediction intervals for correlation effects. To achieve the exact 95% of confidence for the predicted thermal power values, the prediction interval as calculated with the methods mentioned above can be iteratively increased until 95% of the modeled power values are localized within the predicted interval.

As example this correction procedure is also applied to increase the prediction interval of a SST collector model, using 941 test independent data, selected with QDT selection criteria (table 1).

To adjust the confidence or the prediction interval we substituted the E(σ) values, obtained by the regression, with corrected σ_c values (equation 16).

$$\sigma_c = \frac{t_{\alpha/2,n-k,aj}}{t_{\alpha/2,n-k}} E(\sigma) = F_{aj}\ E(\sigma) \qquad (16)$$

It was observed that the σ_c values are very stable. From the 941 independent data points we obtained maximal $t_{\alpha/2,n-k,aj}$ of 1.98 · 1.84 = 3.64 for the QDT3, which is higher than the $t_{\alpha/2,n-k,aj}$ values calculated by [9].

Table 1: Not adjusted and adjusted mean and maximal prediction intervals for the thermal power output of a collector together with its confidence values and the adjusting factors σ_c and F_{adj}

	prediction interval					
	QDT1	QDT2	QDT3	QDT4	SST	unidades
mean	30.08	23.41	17.74	25.58	8.68	W/m²
max.	30.94	25.32	18.68	27.03	9.15	W/m²
$t_{alfa/2,n-k\ (n)}$	1.98	1.98	1.98	1.98	2.16	[-]
σ	13.56	11.53	8.82	12.65	4.08	W/m²
confidence	93.52	89.59	82.47	90.75	29.00	%
	adjusted prediction interval					
	QDT1	QDT2	QDT3	QDT4	SST	unidades
σ_c	15.59	15.45	16.23	15.94	24.48	W/m²
F_{aj}	1.15	1.34	1.84	1.26	6.00	[-]
mean	34.59	31.37	32.65	32.23	52.08	W/m²
max.	35.58	33.92	34.36	34.06	54.89	W/m²
confidence	95.11	95.06	95.20	95.15	95.22	%

As the adjusted prediction and confidence intervals are based on independent measurements, they reflect the real total regression uncertainties whose includes the uncertainties originated by the correlation effects.

The σ_c – values obtained can also be applied to correct the confidence interval (table 2) via the substitution of the estimated E(σ) values in eqn. (12) .

Table 2: Adjusted mean and maximal confidence intervals for the thermal power output of a collector together with its confidences and the adjusting factor σ_c for four QDT and one SST.

	Confidence interval					
	QDT1	QDT2	QDT3	QDT4	SST	units
σ_c	15.59	15.45	16.23	15.94	24.48	W/m²
mean	4.66	6.55	5.60	6.27	18.39	W/m²
max.	9.65	14.64	12.18	12.79	25.76	W/m²
confidence	95.11	95.06	95.20	95.15	95.22	%

5.4 Future adjustments

Assuming similar conditions in future collector tests, the σ_c – values derived here can be also used for additional tests of similar collectors to correct the uncertainty of the mean response.

Using the four σ_c – values given in table 2, we estimate a 99% uncertainty interval [σ_c = (15.8 ± 1.02) W/m^2] for this parameter using the following equation

$$U = u_c\ t = u_p\ t = \sigma(\bar{x})\,t = \frac{\sigma}{\sqrt{n}}\,t\ ; \qquad u_p = \sigma(\bar{x}) = \frac{\sigma}{\sqrt{n}} \tag{17}$$

where the *student – t* value is determined for n = 4 values, using the standard deviation σ of the mean value calculated with the four σ_c – values.

6. Extended uncertainty of yearly solar energy conversion

The energy that a collector can convert by use of the solar radiation is calculated by

$$Q_{mo} = \pm \sum_{i=1}^{n} \dot{Q}_{mo,i}\ \tau_m \tag{18}$$

where i = 1…n are the used power values, τ_m is the time interval over that one power is integrated, that is in the accomplished QDT collector test 5 min, and for a given TMY one hour.

The uncertainty $U(Q_{mo})$ for a $100(1-\alpha)$ confidence interval for the estimated collector energy Q_{mo} of a collector test is calculated with the following equation

$$U(Q_{mo}) = \pm\sum_{i=1}^{n} U(\dot{Q}_{mo,i})\ \tau_m \tag{19}$$

where τ_m is the time in which the mean values of the measurable quantities are taken from the samples of the experiment. It should be pointed out, that the differences of the measured and the modeled energy may have the tendency to decrease for the increased number of values, but not the uncertainty calculated with equation (19). The presented method is therefore to be considered as a conservative estimation method

The extended uncertainty has to consider all uncertainty sources that may appear in a certain process [8]. Assumed that the input data used to compare different collectors, here the *typical metrological year* (TMY), are a reference without uncertainty, we have to consider the regression uncertainties $U(Q)_{regr}$, the uncertainties of the measured collector power and the not corrected systematic uncertainties of the pyranometers used in the test for the yearly energy gain.

6.1 Systematic uncertainties of the pyranometers

The pyranometers are the only transducers with considerable not compensable systematic uncertainties. To assess the uncertainty of the energy gain within a defined period, the uncertainties for the average power output has to be analyzed. As example for the global radiation G = 800 W/m^2 and the diffuse radiation G_d =120 W/m^2 that are used for the plot of the normalized efficiency curve, these systematic uncertainties were calculated by [13] with 29.83 W/m^2 and 9.77 W/m^2. As some of the uncertainty values are expressed in relative and some in absolute values, the same uncertainty budget was used to calculate the corresponding uncertainty for each of radiation intensity appearing within the QDT. The results were used to calculate the uncertainties of the instantaneous collector power values with the equation (20).

$$U(\dot{Q}_{mo,QDT}) = \sqrt{\left(\frac{\partial(\dot{Q}_{mo,QDT})}{\partial(G)}U(G)\right)^2 + \left(\frac{\partial(\dot{Q}_{mo,QDT})}{\partial(G_d)}U(G_d)\right)^2} \tag{20}$$

Considering the radiation data of a QDT as representative for the radiation data of the real application, the mean value of the obtained uncertainty within a collector test using equation (20) is considered as the systematic pyranometer uncertainty, which has to be accounted in the uncertainty budget (table 3).

6.2 Random uncertainties

The random uncertainties of the measured thermal power values $u(\dot{Q}_{me})$ (equation 3, not calculated as value in table 3) was obtained by the following equation.

$$u\left(\dot{Q}_{me}\right)=\sqrt{\begin{aligned}&\left[\frac{\partial\left(\dot{Q}_{me}\right)}{\partial\left(\dot{V}\right)}u(\dot{V})\right]^2+\left[\frac{\partial\left(\dot{Q}_{me}\right)}{\partial\left(\rho_f\right)}u\left(\rho_f\right)\right]^2+\left[\frac{\partial\left(\dot{Q}_{me}\right)}{\partial(C_p)}u(C_p)\right]^2\\&+\left[\frac{\partial\left(\dot{Q}_{me}\right)}{\partial\left(T_{in}\right)}u(T_{in})\right]^2+\left[\frac{\partial\left(\dot{Q}_{me}\right)}{\partial\left(T_{out}\right)}u(T_{out})\right]^2+\left[\frac{\partial\left(\dot{Q}_{me}\right)}{\partial(A)}u(A)\right]^2\end{aligned}} \quad (21)$$

The *standard uncertainties* u are obtained dividing the *brute values* (table 3, 4 e A1) by the elaborated *divisor*, which is dependent onto the assumed distribution of the uncertainty, where the uniform distribution is a more conservative assumption. The *combined uncertainty* u_c is obtained by equation (22) and the *combined degrees of freedom* is obtained by the *Welch-Satterthwaite* equation [8] and the *expanded uncertainty* U is calculated by multiplication of the *combined uncertainty* (eqn. 22) with the *student-t value* that corresponds to the *combined degrees of freedom* [8].

$$u_c = \sqrt{u_1^{\ 2} + u_2^{\ 2} ... u_n^{\ 2}} \quad (22)$$

Table 3: Uncertainty budget for the energy gain estimated with the results of a quasi dynamic test

source of the uncertainties		**random and systematic effects**				
symbol	**description**	**brute values [W/m²]**	**type of the distribution**	**divisor**	**u [Wh/m²]**	**ν**
	Uncertainties obtained by the regression					
UR	Adjusted uncertainties obtaind by the regression	6.55	normal	1.97	3.32	200
	Measuirement of the thermal collector power					
Tin	Measurement of the input temperaure	4.91	normal	2.00	2.46	∞
Tout	Measurement of the output temperaure	4.91	normal	2.00	2.46	∞
VM	Transducer used for the volume flow measurement	1.62	normal	2.00	0.81	∞
Ro	Specific weight of the water	0.15	normal	2.00	0.08	∞
AC	Collector area	0.10	uniform	1.73	0.06	∞
CA	Specific heat of the fluid (water)	0.10	uniform	1.73	0.06	∞
	Systematic uncertainties of the pyranometers					
SIS	Systematic uncertainties of the diffuse and the global radiation	20.14	uniform	1.73	11.63	∞
Cc	**Systematic corrections**					
u_c	**Combined standard uncertainty**		normal		12.61	∞
U	**Expanded uncertainty (95%)**		normal		25.22	

During the performance of the SST a mean radiation of 999 W/m^2 was observed with only ± 38 W/m^2 as standard deviation. Thus for the irradiance affecting the test results, a mean uncertainty U(G) = 39.34 W/m^2, derived for the mean radiation value with the uncertainty calculation method presented in [17] was considered as unique value (see annex 1).

Similar to the QDT case the uncertainty of the instantaneous power output is assessed, here using eqn.(23) instead of (20).

$$U(\dot{Q}_{mo,SST}) = \left(\frac{\partial(\dot{Q}_{mo,SST})}{\partial(G)} U(G) \right) \quad (23)$$

The resulting uncertainty budget for energy gain derived from SST results is given in table 4. These values are calculated for the instantaneous uncertainty values for the power (table 2) using equation 19 under the same assumptions as for QDT case.

Table 4: Uncertainty budget of the steady state test for the mean power value during the test

source of the uncertainties		random and systematic effects				
symbol	**description**	**brute values [W/m²]**	**type of the distribution**	**divisor**	**u [Wh/m²]**	**v**
	Uncertainties obtained by the regression					
UR	Adjusted uncertainties obtaind by the regression	18.39	normal	1.97	9.33	14
	Measurement of the thermal collector power					
Tin	Measurement of the input temperaure	4.91	normal	2.00	2.46	∞
Tout	Measurement of the output temperaure	4.91	normal	2.00	2.46	∞
VM	Transducer used for the volume flow measurement	1.62	normal	2.00	0.81	∞
Ro	Specific weight of the water	0.15	normal	2.00	0.08	∞
AC	Collector area	0.10	uniform	1.73	0.06	∞
CA	Specific heat of the fluid (water)	0.10	uniform	1.73	0.06	∞
	Systematic uncertainties of the pyranometers					
SIS	Systematic uncertainties of the global radiation	24.72	uniform	1.73	14.27	∞
Cc	**Systematic corrections**					
u_c	**Combined standard uncertainty**		normal		17.42	∞
U	**Expanded uncertainty (95%)**		normal		34.84	

In order to understand the uncertainty characteristics of the collector test, the table 5 shows values of the yearly energy gain and its uncertainties. As an example these data are derived using different fixed values of the mean collector temperatures T_m and on the hourly solar meteorological values (solar radiation and the ambient temperature) values of the year 1999 for the city Florianópolis.

Table 5: Energy gain and its uncertainties of SST and QDT outdoor collector tests, calculated for different fixed values of the average collector temperature.

Tm	30	40	50	60	70	°C
Q	958341	745727	557071	394470	256374	Wh/m²
t	3445	2848	2249	1773	1394	hours
Q_{mean}	278.18	261.84	247.70	222.49	183.91	Wh/m²
$U(Q)_{regr(QDT)}$	22565	18654	14731	11613	9131	Wh/m²
$U(Q)_{regr(QDT)}$	2.35	2.50	2.64	2.94	3.56	%
$U(Q)_{QDT}$	86883	71827	56720	44715	35157	Wh/m²
$U(Q)_{QDT}$	**9.07**	**9.63**	**10.18**	**11.34**	**13.71**	**%**
$U(Q)_{regr(SST)}$	63354	52375	41359	32605	25636	Wh/m²
$U(Q)_{regr(SST)}$	6.61	7.02	7.42	8.27	10.00	%
$U(Q)_{SST}$	120024	99224	78355	61771	48567	Wh/m²
$U(Q)_{SST}$	**12.52**	**13.31**	**14.07**	**15.66**	**18.94**	**%**

The row gives the energy gain Q, the operation hours t and the mean energy gain Q_{mean} for the collector used in the tests. It should be noted that the different collector operation temperatures T_m results in different t, in which the absorbed energy of the collector is higher than his heat losses. Q_{mean} shows the mean of the hourly energy converted during the operation hours t. In the second and the third row appear the absolute and the relative uncertainties of the energy, taking into account only the uncertainties from the regression process of the quasi dynamic collector test $U(Q)_{regr(QDT)}$ and for comparison the expanded uncertainties $U(Q)_{QDT}$ of the QDT test. Fourth and fifth row shows the same parameters for the SST. The energy uncertainties $U(Q)_{regr}$ in table 5 are based on the regression uncertainties of 6.55 W/m^2 for the QDT and 38.19 W/m^2 for the SST (see table 2) . The energy uncertainties U(Q) in table 5 are based on the expanded uncertainty of 25.22 W/m^2 obtained for the QDT (table 3) and 34,84 W/m^2 for the SST (table 4).

The higher relative uncertainties of the SST and QDT tests for higher temperatures are caused by the lower mean energy values Q_{mean} at higher operation temperatures of the collector. One can observe that the regression itself gives low contributions to the uncertainties for the QDT for all temperatures, which leads to the conclusion that the collector model and the regression method are expected to be adequate for energy estimations. As can be observed in table 3 and table 4, the highest contributions to the uncertainties are caused by the measurement of the solar radiations. It should be pointed out that the calculations as in table 5 are only of importance, if one intends to calculate the yearly uncertainty, to calculate e.g. the uncertainty of the investment amortization. Other practical items, as e.g. the MBF (meantime between failures) and the yearly climatic variability's that are not discussed in the present article, may play also an important role in this estimation.

7. Discussion and application of the results

The energy uncertainties as result of a collector test can be used for statistic inferences about the stability [15] (i.e. similarity) of the test results. One possibility is to obtain the stability of two regression results, accomplished by testing the stability of the regression coefficients [15, 17] of the model. It is also possible only to compare the collector coefficients or normalized collector curves form different collector tests [18]. As the regression coefficients may be slightly correlated [7, 12], a better stability test can be obtained by the verification of the yearly energy conversion with its uncertainties [12].

7.1 Comparing two collectors with different characteristics

Most common is that a test institute outputs for different collectors tested (e.g. with the QDT), the yearly energy that the different collectors convert. Together with the collector model, a standardized test year (TMY) [14] and a *standardized daily warm water consumption curve* are used as reference for this calculation. To compare the output of different collectors, the uncertainties of the yearly useful energy gain, converted by the collectors, are employable to state if small differences of e.g. two different collectors are originated by the different constructions of the collectors, or may be explained by the random or systematic uncertainties that appear in different tests. As for this comparison the TMY and the water consumption curve work as references, uncertainties of these references have not to be considered. But the energy uncertainty that appears within the test (result of table 3 times the collector test time) has to be considered. The random uncertainties (table 3 and table 4 – *UR until CA*) times those hours of the TMY where positive energy balance is obtained have to be used as well.

7.2 Comparing two different collector tests

Another application is the comparison of two different collector tests (e.g. the SST and the QDT outdoor tests), using the same, or using different collectors. As in section 7.1, it can be assumed here that the TMY and the consumption profile are references and are thus without uncertainties. The uncertainty can be obtained by the same calculation as in the first paragraph.

7.3 Comparing the results of two different test institutions

For the verification of the collector test accomplished by different test institutions, they has to measure the same results (simulated yearly energy), using the same collector (collector with known stable characteristics that is used as reference collector). As above it can be assumed that the TMY and the consumption profile can by applied without considering any uncertainties.

7.4 Comparing the results of two different test institutions, different tests and different collectors

For the national or the international classification of collectors, the different results of collector tests, accomplished by different test institutions, have to be classified to compare different collectors on the market. Uncertainties of the yearly energy gain for a certain TMY gives information about performance differences obtained with the different collectors. Equal to 7.1 it can be assumed that the TMY and the consumption profile can by applied without considering any uncertainties.

8. Conclusion

As discussed in 4.1 to 4.4 the uncertainties including the radiation measurements (table 3 and table 4) that is reducible as already suggested in [13] plays in most of the application cases minor roles (item 4), as for the uncertainty of yearly energy conversion in this cases only the energy uncertainty that appears during the collector test has to be considered. The SST results because of the minor model capability in more uncertainties than the QDT (if the radiation uncertainty is excluded in table 3 and table 4, the extended uncertainty scale down to 9.75 W/m^2 and 19.97 W/m^2). The iteratively obtained correction factors σ_c are very stable and therefore can be applied to other tests of comparable collectors to obtain the uncertainties of the model for independent data.

Regarding the importance of information on the confidence of test results, we suggest that limits of the maximal uncertainties of the regression results should be included into the EN12975 specification.

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Annex 1: Table A1 - Uncertainty budget for the systematic uncertainties of the global radiation pyranometer calculated for the mean value of the global radiation of the presented SST

origin of the uncertainty		systematic corrections with a model	not corrected systematic uncertainties				
symbol	description of the uncertainty		gross value [W/m²]	distribution form	divisor	u [W/m²]	ν
Cal	Calibration uncertainty		30.27	normal	2.00	15.13	∞
DtPa	Drift over time (change / year)		5.00	uniform	1.73	2.88	∞
Rd	Directional response (as a function of the azimuth and the zenith angle)		14.99	uniform	1.73	8.65	∞
OS I	Offset originated by the thermal radiation		7.00	uniform	1.73	4.04	∞
OS II	Offset originated by the temperature change		2.00	uniform	1.73	1.15	∞
Dter	Temperature dependence of the sensitivity		9.99	uniform	1.73	5.77	∞
NL	Non linearity		6.50	uniform	1.73	3.75	∞
ReE	Spectral response		5.00	uniform	1.73	2.88	∞
MIn	Tilt response		2.00	uniform	1.73	1.15	∞
DtS	Long time drift of the measuring system		0.32	uniform	1.73	0.18	∞
A/D	Error of the analog to digital converter of the measuring unit		0.02	uniform	1.73	0.01	∞
	Combined standard uncertainty			**normal**		**19.67**	∞
	Expanded standard uncertainty			**normal**		**39.34**	

source of the uncertainty		systematic corrections with a model	not corrected systematic uncertainties				
symbol	description of the uncertainty		gross value [W/m²]	distribution form	divisor	u [W/m²]	ν
Cal	Calibration uncertainty		22.32	normal	2.00	11.16	∞
DtPa	Drift over time (change / year)		4.00	uniform	1.73	2.31	∞
Rd	Directional response (as a function of the azimuth and the zenith angle)		10.00	uniform	1.73	5.77	∞
OS I	Offset originated by the thermal radiation		7.00	uniform	1.73	4.04	∞
OS II	Offset originated by the temperature change		2.00	uniform	1.73	1.15	∞
Dter	Temperature dependence of the sensitivity		8.00	uniform	1.73	4.62	∞
NL	Non linearity		6.50	uniform	1.73	3.75	∞
ReE	Spectral response		4.00	uniform	1.73	2.31	∞
MIn	Tilt response		1.60	uniform	1.73	0.92	∞
DtS	Long time drift of the measuring system		0.32	uniform	1.73	0.18	∞
A/D	Error of the analog to digital converter of the measuring unit		0.02	uniform	1.73	0.01	∞
	Combined standard uncertainty			**normal**		**14.92**	∞
	Expanded standard uncertainty			**normal**		**29.83**	