COST ASSESSMENT OF AN OPTIMIZED SOLAR-ASSISTED WATER EJECTOR COOLING CYCLE WITH A BOOSTER USING CO_2 AS WORKING FLUID

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Abstract

The present work describes an economic analysis and optimization of a combined ejector-vapor compression refrigeration cycle assisted by solar energy. Two environmentally friendly working fluids are used: water in the ejector sub-cycle, and CO_2 in the vapor compression sub-cycle. The conjugation of the steam-ejector cycle with the CO_2 booster is proposed in order to operate the CO_2 cycle with lower pressure, thus increasing the coefficient of performance (COP). Through the economic analysis the solar collector area and the intercooler temperature are optimized using the f-$\bar{\phi}$ -chart method. Also, a second law analysis was carried out to determine the optimum operation condition for a given exergetic efficiency. The optimization was performed considering the operation of an icemaker for a location in the Amazon River area for given specific cost assessments.

1. INTRODUCTION

Global warming has encouraged researchers to develop environmentally friendly energy systems as a measure to achieve substantial reduction of greenhouse gases emissions. Refrigeration and air-conditioning represent large electricity consumption, especially in tropical regions like Brazil. Concerning the potential impact of industrial refrigerants on earth's atmosphere and their impact on the global climate change, natural refrigerants like water and CO_2 are considered a workable option (Calm, 2008) due its stability and abundance in nature. The challenge developing applications of these refrigerants using solar energy is to achieve systems which are economically competitive with traditional vapor compression cycles.

A solar assisted refrigeration cycle using a booster (auxiliary compressor), proposed by Sokolov and Hershgal (1993), has been studied in last decades. The advantage of this cycle is a substantial increment on cycle's COP, compared with an ejector cycle at the same sink temperatures. Regarding this model, Colle and Vidal (2003) and Vidal *et al.* (2006) presented a conjugated cycle using two different refrigerants: R134a and R141b. Both presented the optimization conditions according the P_1-P_2 method (Brandemuhel e Beckman, 1979), where an hourly simulation is necessary. The authors checked all the equations contained in the paper of Colle and Vidal (2003), and identified several errors and mistakes in the basic equations, which are corrected in the present analysis. Since the availability of validated meteorological data in developing countries is limited, Colle *et al.* (2008) validated the f-$\bar{\phi}$ -chart method (Klein and Beckman, 1979) for the particular case of a solar assisted ejector cycle. This validation allows proceeding with an economic analysis using the solar fraction, calculated through the f-$\bar{\phi}$ -chart method, as a function of monthly means of incident solar radiation.

The present work presents an economic analysis of an icemaker for a location in the Amazon River area by using the f-$\bar{\phi}$ -chart method to correlate solar collectors' parameters with the monthly means of solar radiation incident on Manaus city. The refrigeration cycle is configured by an ejector and mechanical compression sub-cycles, where H_2O and CO_2 are used as working fluids. Hence the optimization carried out considers two variables: solar collector area, and intercooler operation temperature.

Cycle Description

Figure 1 shows the arrangement of the combined cycle. It consists of a solar heating system, an ejector refrigeration system (H_2O) and a mechanical compression system (CO_2). The use of these refrigerants has environmental advantages, besides the obvious cost advantage of water and the possibility to reach lower temperatures by using CO_2. The connection between the two refrigeration sub-cycles is an intercooler, which serves as evaporator for the ejector system, and as condenser for the mechanical compression system. Its working temperature T_e is set between the condenser temperature T_c and the evaporator temperature of the CO_2 booster, T_r.

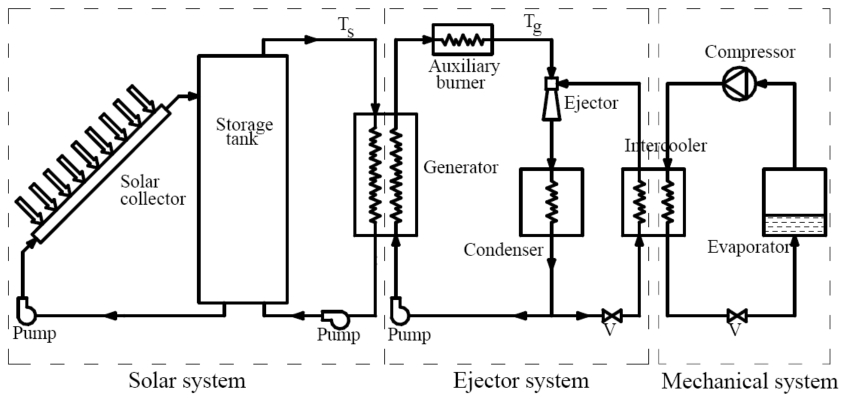

Figure 1. Solar assisted water ejector with CO_2 booster

The working principle of the cycle is as follows: solar collectors supply heat to a vapor generator, which operates as the heat source for ejector cooling cycle. Water evaporates in the vapor generator at temperature T_g. An auxiliary heating system is placed if the amount of heat provided by the solar heating system is unable to satisfy the thermal load requirements. The combined operation of solar and auxiliary heating systems guarantees a proper steady-state flow rate of refrigerant. The steam flows though the convergent-divergent nozzle of the ejector. As it enters the mixing section, a low pressure region is caused by the expansion, which induces the secondary steam flow from the intercooler. The primary and secondary steam flows are mixed in the ejector, and the combined stream undergoes a pressure recovery process at the ejector's diffuser. The mixture of both steams flows to a condenser and loses heat at a temperature T_c. After the condenser, the flow splits into primary, which is pumped back to the vapor generator, and secondary, which flows to the intercooler at T_r after passing through an expansion valve. The ratio of primary to secondary nozzle cross section areas of the ejector is designed in order to achieve the maximum flow ratio in the evaporator, for a given flow ratio of the primary stream. In the mechanical compression system the compressed CO_2 is condensed in the intercooler, where the latent heat released is used to vaporize the water. The condensate undergoes a pressure reduction in a throttle valve and then enters the evaporator where it produces the necessary cooling effect.

2. ECONOMIC EVALUATION AND OPTIMIZATION

Ejector's cycle COP is defined by the ratio between the heat transferred at intercooler (Q_e) and the heat transferred in vapor generator (Q_g), as follows:

$$COP_{ej}(T_e, T_c, T_g) = \frac{Q_e}{Q_g} \tag{1}$$

This COP is estimated using the thermodynamic model presented by Sun (1997) and Eames *et al.* (1995). For a mechanical vapor compression cycle, the COP is defined as:

$$COP_m(T_r, T_e) = \frac{Q_r}{W_m} \tag{2}$$

where Q_r is the refrigeration load and W_m is the mechanical power consumption by the compressor.

Defining the COP for the combined cycle as:

$$COP = \frac{Q_r}{Q_g + W_m}, \tag{3}$$

then the expressions above leads to:

$$COP(T_r, T_e, T_c, T_g) = \frac{COP_{ej} COP_m}{1 + COP_m + COP_{ej}} \tag{4}$$

In order to estimate the economic performance of the combined cycle, it is compared with a traditional mechanical compression cycle operating at the same sink temperatures. Concerning the environmental issues described before, the cycle was compared with a transcritical CO_2 cycle, whose COP is expressed by:

$$COP_M(T_r, T_c) = \frac{Q_r}{W_M} \tag{5}$$

Regarding the definitions of Klein and Beckman (1979), the lifetime cost savings function for a combined ejector-mechanical cycle is defined as the difference between the total capital cost for a fully mechanical system and for the proposed cycle. This function according Colle *et al.* (2004) is defined as follows:

$$LCS = P_1 Q_r \Delta t \left[C_{E1}\left(\frac{1}{COP_M} - \frac{1}{COP_m}\right) - C_{F1}\frac{(1-f)}{COP}\right] - P_2\left(C_A A_C + C_M - C_m - C_{EJ} - C_E\right), \tag{6}$$

where P_1 is the present worth factor for a uniform series of expenses in electricity or natural gas, consumed by the auxiliary system. C_{E1} and C_{F1} are the specific cost of electricity and natural gas, respectively, at the first year of analysis. P_2 is an economic factor, described by Brandemuehl and Beckman (1979), which is composed by system's financial cost, depreciation and other minor capital cost. C_A is the solar collector cost per unit of area, C_m is the capital cost of CO_2 booster, C_{EJ} is the capital cost of ejector system (including thermal reservoir, steam generator and intercooler), C_E is total cost of equipment which is independent of collector area, A_C is the collector area and f is the annual solar fraction, calculated by the f-$\bar{\phi}$ -chart method considering $T_{min} = T_g$ as suggested by Colle *et al.* (2008).

The annual solar fraction is expressed in terms of the monthly refrigeration load (Q_{ri}) and the annual refrigeration load (Q_r) as follows:

$$f = \sum_{i=1}^{12} \frac{f_i Q_{r_i}}{Q_r} \tag{7}$$

According the methodology of Klein and Beckman (1979) the monthly solar fraction is expressed by the following correlation,

$$f_i = \bar{\phi}_{\max} Yi - 0.0015\left(e^{3.85 f_i} - 1\right)\left(1 - e^{0.15 X_i}\right) R_s^{\;0.76} \tag{8}$$

where $\bar{\phi}_{\max}$ is the maximum daily utilizability and R_s is the ratio of standard storage heat capacity per unit of collector area, assumed unity for this analysis. The parameters X_i and Y_i are modified to include the COP of the refrigeration system, as follows:

$$Y_i = A_C F_R(\tau\alpha)_n \left[\frac{(\overline{\tau\alpha})}{(\tau\alpha)_n}\right] \overline{H}_{T_i} N_i \frac{COP}{Q_{r_i}} \tag{9}$$

$$X_i = A_C (F_R U_L) 100 \Delta t_i \frac{COP}{Q_{r_i}} \tag{10}$$

where $F_R U_L$ and $F_R(\tau\alpha)_n$ are the collector efficiency coefficients, and $\overline{H}_{T_i}$ is the monthly average daily radiation on collector surface.

Alternatively, equation (6) can be rewritten as:

$$\ell = \alpha_E \left(\frac{1}{COP_M} - \frac{1}{COP_m} \right) - \frac{\alpha_F (1-f)}{COP} - a_c + \frac{d}{C_A} \tag{11}$$

where $\ell = \dfrac{LCS}{P_2 C_A Q_r \Delta t}$,

$\alpha_E = \dfrac{P_1 C_{E1}}{P_2 C_A}$,

$\alpha_F = \dfrac{P_1 C_{F1}}{P_2 C_A}$,

and $d = \dfrac{C_M - C_m - C_{EJ} - C_E}{Q_r \Delta t}$

The solar fraction (f) depends on area A_c, efficiency factors ($F_R U_L$, $F_R(\tau\alpha)_n$), COP(T_r, T_e, T_c, T_g) and temperature $T_{min.}$ Thus, considering the economic viability of the system for the cases whit $\ell \geq 0$ ensues,

$$\alpha_E \left(\frac{1}{COP_M} - \frac{1}{COP_m} \right) + \frac{d}{C_A} - a_c \geq \frac{\alpha_F (1-f)}{COP} \geq 0 \tag{12}$$

The above inequality shows the existence of a maximum specific area a_{max}, defined as:

$$a_{\max} = \alpha_E \left(\frac{1}{COP_M} - \frac{1}{COP_m} \right) + \frac{d}{C_A} \tag{13}$$

Deriving equation (11) with respect to a_c, obtains the following expression:

$$\alpha_F \frac{\partial f}{\partial a_c} = COP \tag{14}$$

Thus for the limit case of $\ell = 0$ equation (11) can be rewritten as:

$$a_{\max} - a_c = \frac{\alpha_F (1-f)}{COP} \tag{15}$$

Replacing α_F from equation (14) into equation (15), it leads to:

$$(a_{\max} - a_c) \frac{\partial f}{\partial a_c} = 1 - f \tag{16}$$

Instead of the expression in equation (11), equation (6) can be rewritten as:

$$\ell = \frac{\alpha_E}{COP_M} - \psi - a_c + \frac{d}{C_A} \ , \qquad (17)$$

where $\psi = \dfrac{\alpha_E}{COP_m} + \dfrac{\alpha_F(1-f)}{COP}$.

Thus, deriving equation (11) with respect to the temperature T_e, leads to

$$\frac{\partial \ell}{\partial T_e} = -\frac{\partial \psi}{\partial T_e} = \frac{\alpha_E}{COP_m^2}\frac{\partial COP_m}{\partial T_e} + \frac{\alpha_F}{COP}\left[\frac{(1-f)}{COP}\frac{\partial COP}{\partial T_e} + \frac{\partial f}{\partial T_e}\right] \qquad (18)$$

Hence, making the equation above equal to zero, the following expression is found:

$$\alpha_E \frac{\partial COP_m}{\partial T_e} = -\alpha_F \frac{COP_m^2}{COP}\left[\frac{\partial f}{\partial T_e} + \frac{(1-f)}{COP}\frac{\partial COP}{\partial T_e}\right] \qquad (19)$$

Equations (16) and (19) can be solved in terms of a_c and T_e for a given pair (α_F, α_E). Therefore, replacing α_F from equation (14) into (15) through a_{max}, is possible to express α_E as function of a_c. Hence, curves of $\ell = \partial\ell/\partial a_c = 0$ on coordinates α_F and α_E can be plotted. Similarly, replacing α_F in equation (10), it is also possible to express α_E as function of a_c and plot curves of $\partial\ell/\partial T_e = 0$, for a constant T_e.

Icemaker

In order to illustrate the economic analysis described before a numeric example is analyzed. Considering the installation of a 3TR (10.55 kW) icemaker in Manaus (Amazon River), the operation temperatures of the system are listed below:

T_r = -5 °C $\qquad$ T_c = 35 °C $\qquad$ T_g = 120 °C

Solar collector used is evacuated tube type, with efficiency parameters provided by manufacturer. Capital cost of refrigeration cycles were estimated using in ASPEN Economic Evaluator (2009). The economic life considered was 20 years, with a discount rate of 8%, and inflation rate of 3%.

Regarding the considerations above, Figure 2a shows the root a_c for the corresponding a_{max} value. At this figure the curve h is defined as:

$$h = (a_{\max} - a_c)\frac{\partial f}{\partial a_c} \qquad (20)$$

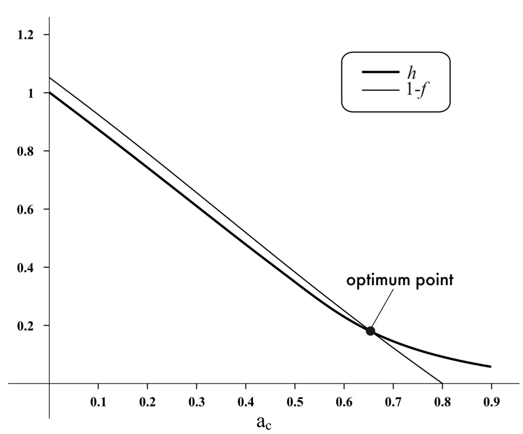

Figure 2a. Solution for the specific area a_c

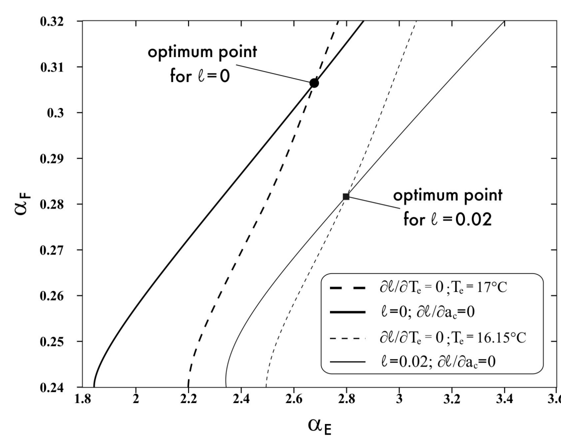

Figure 2b. Curves of ℓ and $\partial\ell/\partial T_e$ for different numeric values of d/C_A

Figure 2b illustrates the feasibility diagram for the icemaker, where the region below the curve ℓ = 0 corresponds the economically viable points.

The cost function ψ has two bound limits, the first one for $T_e=T_r$ which is given by

$$\psi(T_r)=\frac{\alpha_F(1-f_r)}{COP_{ej}(T_r,T_c,T_g)} \tag{21}$$

where f_r is the value of the solar fraction f, evaluated at $T_e=T_r$. The second bound for $T_e=T_c$, is given by

$$\psi(T_c)=\frac{\alpha_E}{COP_m(T_r,T_c)} \tag{22}$$

Let us define the cost ratio $\lambda_c=\alpha_E/\alpha_F$, and

$$\lambda_r=\frac{(1-f_r)COP_m(T_r,T_c)}{COP_{ej}(T_r,T_c,T_g)}, \tag{23}$$

The following three cases arise from equations (21), (22), and (23),

i. $\lambda_c = \lambda_r$, which leads to $\psi(T_c) = \psi(T_r)$,

ii. $\lambda_c < \lambda_r$, which leads to $\psi(T_c) > \psi(T_r)$, and

iii. $\lambda_c > \lambda_r$, which leads to $\psi(T_c) < \psi(T_r)$

Figure 3 shows the shape of the cost function ψ for different intercooler temperatures. Also, the Carnot cycle limit was plotted. It is observed a strong dependence on the intercooler temperature, which is dependent on the ratio λ_c.

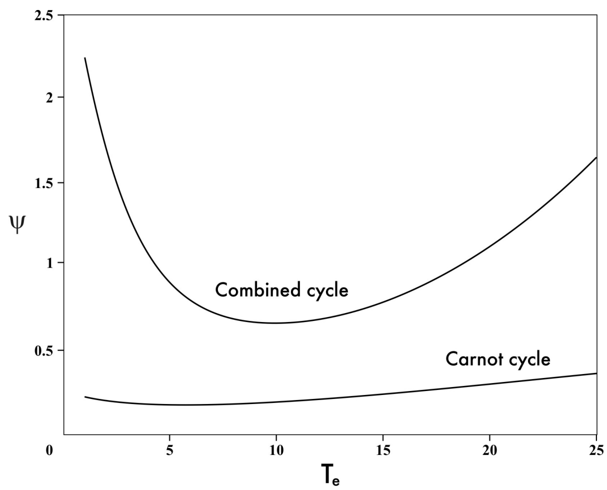

Figure 3. Cost function comparison of the combined and Carnot Cycles

3. SECOND LAW ANALISYS

The optimization methodology described in previous section was carried out regardless the second law of thermodynamics. However, the relationship between economic parameters and the thermodynamic performance of system's components have particular interest on optimization of operation conditions.

For the combined cycle proposed, according to the theory presented in Kotas (1995), the entropy generation of the whole cycle is expressed as follows:

$$S_{gen}=\frac{Q_c}{T_c}-\frac{Q_g}{T_g}-\frac{Q_r}{T_r} \tag{24}$$

where

$$Q_c=Q_r+Q_g+W_m \tag{25}$$

Using equations (2), (3) and (25), equation (24) can be rewritten as:

$$S_{gen} = Q_r \left[\frac{1}{COP}\left(\frac{1}{T_c} - \frac{1}{T_g}\right) + \frac{1}{T_c} - \frac{1}{T_r} + \frac{1}{T_g COP_m} \right] \tag{26}$$

From the definition of irreversibility associated to equation (24), as defined by Kotas (1995), referred to the environment temperature T_o, according to equation (26), is given by

$$I = T_o Q_r \left[\frac{1}{COP}\left(\frac{1}{T_c} - \frac{1}{T_g}\right) + \frac{1}{T_c} - \frac{1}{T_r} + \frac{1}{T_g COP_m} \right] \tag{27}$$

Thus, the exergetic efficiency is defined by

$$\eta_{ex} = 1 - \frac{I}{\sum E_{Qin} + W_m} \tag{28}$$

where $\sum E_{Qin}$ is the exergy sum of the heat input.

The overall input exergy for the whole system is proven to be given by the following expression.

$$\sum E_{Qin} + W_m = Q_r \left[\left(\frac{1}{COP} - \frac{1}{COP_m}\right)\left(\frac{T_g - T_o}{T_g}\right) + \frac{1}{COP_m} \right] \tag{29}$$

The specific irreversibility is defined as $\phi = I/Q_r$. From equation (27), ϕ can be written as follows

$$\phi = T_o \left[\frac{1}{COP}\left(\frac{1}{T_c} - \frac{1}{T_g}\right) + \frac{1}{T_c} - \frac{1}{T_r} + \frac{1}{T_g COP_m} \right] \tag{30}$$

From equation (28) and the definition of ϕ, the exergetic efficiency can be written as $\eta_{ex} = 1 - \phi Q_r / (\sum E_{Qin} + W_m)$. Replacing the denominator by its expression given by equation (29) in the former equation, the exergetic efficiency can be written as follows

$$\eta_{ex} = 1 - \phi \Big/ \left[\left(\frac{1}{COP} - \frac{1}{COP_m}\right)\left(\frac{T_g - T_o}{T_g}\right) + \frac{1}{COP_m} \right] \tag{31}$$

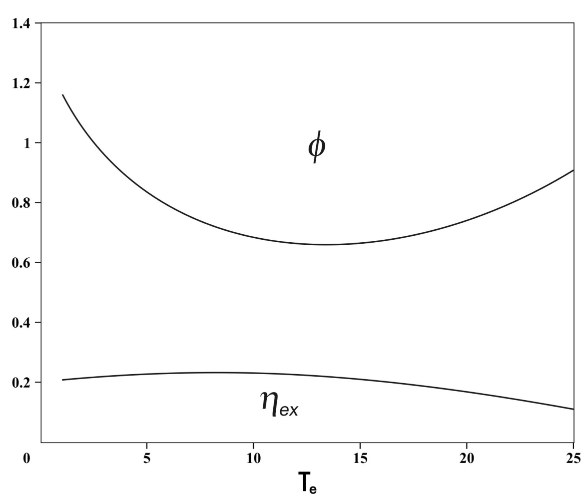

Figure 4. Exergetic efficiency and irreversibility functions

Figure 4 shows the curves of exergetic efficiency and the specific irreversibility, computed from equations (30) and (31), for different intercooler temperatures. From this figure is inferred the existence of an optimum value of these functions

Constrained Optimization

It is verified that the optimum point achieved by the cost function ψ is significantly different from the minimum point achieved by the irreversibility function ϕ. Therefore it makes sense to optimize the cost function ℓ for constant values of ϕ. The cycle irreversibility defined in equation (30) is function of the intercooler temperature T_e. Therefore from the mentioned equation, the intercooler operating temperature can be found for each specified value of ϕ.

A constrained optimization of function ℓ in terms of collector area and intercooler temperature is carried out by applying the method of Lagrange multipliers. Hence, let us define the auxiliary function L as follows:

$$L = \ell + \lambda_\ell(\phi - \phi_o) \tag{32}$$

for a specified value of ϕ_o or I_o. Taking the derivative of equation (32) with respect to a_c, equation (14) is recovered. Nevertheless, taking the derivative of the same equation with respect to T_e, the following equation holds,

$$\frac{\partial \ell}{\partial T_e} + \lambda_\ell \frac{\partial \phi}{\partial T_e} = 0 \tag{33}$$

In terms of function ψ, the following equation holds,

$$-\frac{\partial \psi}{\partial T_e} + \lambda_\ell \frac{\partial \phi}{\partial T_e} = 0 \tag{34}$$

Thus according to the equation above, for each numerical value of η_{ex}, T_e can be determined from equations (30) and (31), and then a_c can be found from equations (15) and (16). Using the calculated values of T_e and a_c, the Lagrange multiplier is obtained from equation (34) as follows,

$$\lambda_\ell = \frac{\partial \psi}{\partial T_e} \Big/ \frac{\partial \phi}{\partial T_e} \tag{35}$$

Therefore, curves for constant value of ℓ as function of a_c and η_{ex} (or T_e) can be plotted. Once the Lagrange multiplier λ_ℓ is calculated, the optimum sensitivity coefficient for the optimum value of ℓ is know to be expressed by the following partial derivative

$$\lambda_\ell = \frac{\partial \ell_{opt}}{\partial \phi_o} \tag{36}$$

The equations above can be used straightforward to carry out the numerical evaluation of the constrained optimum.

4. CONCLUSIONS

The present work reports a basic economic analysis of a solar assisted ejector cooling cycle with a booster using two environmentally friendly working fluids: water and CO_2. This approach is useful to determine the conditions under which, the proposed cycle is economically viable.

The methodology proposed was used to evaluate the operation of an icemaker in Amazon River. For this example, considering d/C_A=0, the optimum collector area is 102 m^2. From Figure 2.b is o observed that higher values of d/C_A will increase the viable region at the diagram.

Numerical example shows different optimum solutions for temperature for life cycle cost savings function and for exergetic efficiency function. This result is expected since an optimum economic design; in general is not equivalent to an optimum thermodynamic design (Kotas, 1995).

It worth mention that the concept of the system proposed here, for ice production in the Amazon region, has considered the condensation process by using air condensers. A more attractive alternative is to consider condensation process by using water from rivers as condensing fluid. Since the temperature of the water in the Amazon region is lower than 20°C, liquid condenser enables one to reduce the condenser temperature to 25°C. Lower condenser temperatures increase the COP of the system, and therefore reduce the required collector area.

The extension of the present paper to the case of condensers with liquid as cooling fluid is being carried out, for both unconstrained and constrained optimization, by considering other cost scenarios.

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