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Solar Energy 76 (2004) 125–133
SOLAR ENERGY
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Upper bounds for thermally driven cooling cycles optimization derived from the f–$\bar{\phi}$ chart method

Sergio Colle [a,*], Humberto Vidal [b,1]

[a] LABSOLAR, Department of Mechanical Engineering, Federal University of Santa Catarina, P.O. Box 476, Florianópolis, SC 88040-900, Brazil
[b] Department of Mechanical Engineering, University of Magallanes, P.O. Box 113-D, Punta Arenas, Chile

Received 17 January 2003; received in revised form 23 July 2003; accepted 28 July 2003

Abstract

An analytical approach for the economical evaluation and optimization of absorption and an ejector cooling cycle is presented. The f–$\bar{\phi}$ chart method is used here to correlate the basic design parameters of the solar system with the system cooling cycle performance. The optimization is carried out by using the life cycle cost savings function as the objective function. This function is expressed in terms of the capital cost and the operating cost, the later expressed in terms of the solar fraction f. The optimization is carried out for a lithium-bromide chiller and an ejector cooling cycle with R11 as working fluid. A numerical example is presented to compare the optimum bound regions for both cycles, in terms of the capital costs respective to each system. The Carnot cycle limit is also determined. Upper bounds for economical feasibility in terms of the costs of the auxiliary energy and electric energy are also set down. The design approach presented here is convenient to determine the optimum conditions in terms of the monthly means of the global radiation incident on a horizontal surface.

1. Introduction

The increasing cost of electrical energy worldwide and decreasing production cost of flat plate collectors in many countries give rise to investigate new concepts of thermally driven cycles. The ejector cycle assisted by solar energy in many circumstances has been proved to be economically attractive and has been shown by Sokolov and Hershgal (1993), Medina (1997) and Medina and Colle (2001). The absorption cycles are usually more expensive than mechanical compression cycles, because the complexity and the size of the construction of the former, and also because of the size of the cooling tower. Absorption systems assisted by solar energy are in general more expensive than ejector systems, because the former usually requires higher operating temperatures of the heat supply and also because the later are much simpler in construction. On the other hand, the coefficient of performance of ejector cycles is usually smaller than the coefficient of performance of absorption cycles. This advantage of the absorption cycles can be shown to be of major importance in the economical feasibility in favor of absorption system. Presently both, absorption and ejector systems are seldom competitive with the cooling cycles based on mechanical compression, particularly in the case the heat is provided by solar energy. The optimization with respect to the solar collector area and the operating temperatures are therefore needed, in order to investigate the limits under which the thermally driven cycle (TDC) may become economically competitive with the mechanically driven cycle (MDC).

The f–$\bar{\phi}$ chart method described in Klein and Beckman (1979) is proposed to estimate the long term performance of solar heating systems, for designing process heat and power systems, for which the

* Corresponding author. Tel.: +55-48-2342161/5166; fax: +55-48-3317615.
E-mail addresses: colle@emc.ufsc.br (S. Colle), rvidal@ona.fi.umag.cl (H. Vidal).
[1] Tel.: +56-61-207000; fax: +56-61-232284.

0038-092X/$ - see front matter
doi:10.1016/j.solener.2003.09.001

Nomenclature

A_c collector area, m^2
a_c collector specific area per refrigeration load, $m^2\,GJ^{-1}$
C_A collector cost per unitary area, US\$ m^{-2}
C_{CT} capital cost of the cooling tower, US\$
C_E cost independent of collector area, US\$
C_{EL} capital cost of the electric-mechanical compression cooling system, US\$
C_{E1} cost of electric energy in the first year of the period of economical analysis, US\$ $kW\,h^{-1}$
C_{F1} cost of the auxiliary energy in the first year of the period of economical analysis, US\$ $kW\,h^{-1}$
COP coefficient of performance of the thermally driven cycle, dimensionless
C_{TH} capital cost of the thermally driven cooling system, US\$
f_i fraction of the monthly load supply by solar energy, dimensionless
F_R collector overall heat removal efficiency factor, dimensionless
G_{SC} solar constant, W/m^{-2}
$\overline{H}$ monthly average daily total solar radiation on a horizontal surface, $MJ\,m^{-2}$
$\overline{H}_o$ monthly average daily extraterrestrial solar radiation, $MJ\,m^{-2}$
$\overline{H}_T$ monthly average daily total solar radiation on the collector surface, $MJ\,m^{-2}$
K_T daily clearness index on the collector surface, dimensionless
$\overline{K}_T$ ratio of the monthly average total to the monthly average extraterrestrial radiation on a horizontal surface, dimensionless
LCS life cycle savings, US\$
N numbers of days in a month
P_1 ratio of life cycle fuel savings to first year fuel energy cost, dimensionless
P_2 ratio of owning cost to initial cost, dimensionless
Q_c condenser heat transfer, GJ
Q_g generator heat transfer, GJ
Q_L annual amount of heat required to drive the cooling cycle, GJ
Q_r cooling demand, GJ
Q_s solar heat, GJ
T temperature, °C
$\overline{T}_a$ monthly average ambient temperature, °C
T_{min} minimum useful energy temperature, °C
U_L collector overall energy loss coefficient, $W\,m^{-2}\,K^{-1}$
$\overline{R}$ ratio of the monthly average daily total radiation on a tilted surface to that on a horizontal surface, dimensionless
$\overline{R}_b$ ratio of the average daily beam radiation on the tilted surface to that on a horizontal surface, dimensionless
$R_{b,n}$ ratio of beam radiation on the tilted surface to that on a horizontal surface at solar noon, dimensionless
R_n ratio of radiation on a tilted surface to that on a horizontal surface at noon, dimensionless
$r_{d,n}$ ratio of the diffuse solar radiation at solar noon to the daily total radiation on a horizontal surface, dimensionless
$r_{t,n}$ ratio of the radiation at solar noon to the daily total radiation on a horizontal surface, dimensionless
R_S ratio of the standard storage heat capacity per unit of collector area (350 $kJ\,m^{-2}\,K^{-1}$) to the actual storage capacity, dimensionless
X' modified sensibility factor of the thermal losses, dimensionless
$\overline{X}_c$ monthly average critical radiation ratio, dimensionless
$\overline{X}_{c,min}$ minimum monthly average critical radiation ratio, dimensionless
Y ratio of the absorbed solar energy to the cooling load, dimensionless

Greek symbols
α_E cost parameter proportional to the operational cost, $m^2\,kW\,h^{-1}$
α_F cost parameter proportional to the auxiliary energy cost, $m^2\,GJ^{-1}$
β tilt angle of solar collector

δ	solar declination angle
Δt	total number of seconds in the month considered
ϕ	latitude angle
$\bar{\phi}_{\max,i}$	maximum monthly average daily utilizability, dimensionless
ρ_g	reflectance of the ground surrounding the collectors, dimensionless
$(\tau\alpha)_n$	transmittance–absorptance product for radiation at normal incidence, dimensionless
$(\overline{\tau\alpha})_i$	monthly average energy transmittance–absorptance product, dimensionless
ω	hour angle
ω_s	sunset (or sunrise) hour angle on a horizontal surface
ω'_s	sunset (or sunrise) hour angle on a tilted surface

Subscripts

a	ambient
abs	absorption
c	condenser
e	evaporator
ej	ejector
g	generator
i	i-th month
L	load
n	noon
T	tilted surface
TH	thermal system

thermodynamic cycle efficiency is independent of the heat supply temperature. This method is useful for designing absorption systems assisted by solar energy, because the coefficient of performance of the absorption cycle is nearly independent of the temperature of the heat supply. This method requires basically the monthly means of the global solar radiation incident on the horizontal surface. Monthly means, rather than hourly totals of global radiation are usually available in databases of national weather services of many countries. On the other hand, the mapping of monthly averages of solar global radiation incident on the horizontal surfaces derived from satellites by statistical and physical models has been successfully carried out for mapping continental solar energy resources, as reported in Stuhlmann et al. (1990), Pereira et al. (1996), and Beyer et al. (1997). The f–$\bar{\phi}$ chart method is particularly applicable in practical situations where the design parameters sensibility with the economic figures of merit is needed. Presently, low cost computation resources available, turns the full time simulation to be mostly attractive, as shown by the everyday increasing number of users of TRNSYS package. An additional advantage of use of the f–$\bar{\phi}$ chart method in the present paper is basically due to its simple analytical formulation, which enables one to get straightforward derivation of the optimum economical design parameters. Moreover, the economical optimum design of the solar system may be considered as a first step, before going to the full simulation, in order to determine the real performance of the system, as well as the critical conditions of operations.

The extension of the f–$\bar{\phi}$ chart method for ejector cooling cycles is suggested for the solar assisted cooling cycles shown in Fig. 1. In this system, the burner supplies auxiliary heat whenever the energy supplied by the solar system at temperatures greater than $T_{\min}$ is not sufficient to drive the ejector cycle. The minimum heat supply temperature can be set equal to the condenser temperature of the ejector cooling cycle, while the minimum temperature is assumed to be the generator temperature, for the case of the absorption cycle. In spite of the fact the f–$\bar{\phi}$ chart method has not been validated for temperature dependent system efficiency or performance in so far, it is used here also to optimize the lifetime cost saving of ejector systems. The coefficient of performance of the ejector cycles is dependent on the generator temperature of the primary flow of the ejector. The governing equations related to the ejector cycle can be found in Sokolov and Hershgal (1993), Medina (1997), Sun (1997), Cizungu et al. (1999), and Huang et al. (1999). These equations are used in order to determine the correlation of the coefficients of performance of the ejector as a function of the operating temperatures. Since in the present analysis the generator temperature, the condenser temperature and the evaporator temperature are assumed to be constant, the coefficient of performance is thus constant. Therefore the f–$\bar{\phi}$ chart method can be used with no restrictions. As in the absorption cycle case analyzed by Klein and Beckman

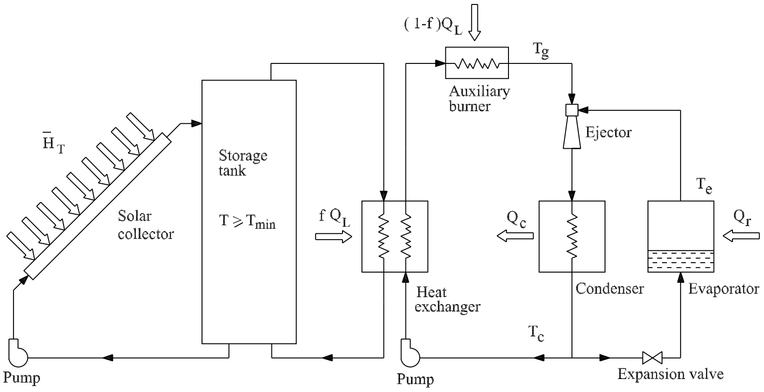

Fig. 1. Ejector cooling cycle assisted by solar energy.

(1979), the cooling load is assumed to be constant for each month.

2. Economical analysis and optimization

Several methods for economical analysis of solar systems are presented in detail in Duffie and Beckman (1991). Among them, the life cycle cost savings method (LCS) has shown to be a simple and practical method to derive the optimization function in terms of the basic costs of the system, the load, and the design parameters. In the present analysis the cost function LCS will be expressed in terms of the solar collector area, the solar fraction f, the capital cost and the auxiliary energy cost.

Let Q_L be the annual amount of heat required to drive the cooling cycle for a specified annual cooling capacity Q_r. Neglecting the work of the liquid pump of the ejector cycle shown in Fig. 1, the coefficient of performance of the thermally driven cycle is then defined by

$$\mathrm{COP} = Q_r/Q_L \tag{1}$$

As shown in Fig. 1, Q_L is the sum of the heat supplied by the auxiliary burner and the heat supplied by the solar thermal system connected to the thermally driven system. The fraction of energy saved by the solar system is defined by $f = Q_S/Q_L$ so that the heat saved at the burner for a year period is $Q_S = fQ_L$. According to the method $P_1 - P_2$, given in Duffie and Beckman (1991), the cost savings related to the period of economical analysis of N_e years, is expressed as the present value of the savings due to the operating cost minus the present value corresponding to the capital costs. In the present analysis the life cost savings function LCS is expressed as

$$\mathrm{LCS} = \frac{P_1 C_{E1} Q_r}{\mathrm{COP}_{el}} - \frac{P_1 C_{F1} Q_r (1-f)}{\mathrm{COP}} - P_2(C_A A_C + C_E) - P_2(C_{TH} - C_{EL} + \Delta C_{CT}) \tag{2}$$

where C_{F1} is the cost of the auxiliary energy (US\$/kW h), C_{E1} is the cost of electric energy (US\$/kW h), C_A is the collector cost per unitary area (U\$/m^2), C_E is the installation and other minor cost of the collector system (US\$), C_{EL} is the capital cost of the MDC (US\$) with the same cooling capacity Q_r (GJ), C_{TH} is the capital cost of the TDC (US\$), and COP_{el} is the coefficient of performance of the MDC moved by electric energy. For the ejector cycle, COP is a function of the vapor generator temperature T_g, the condenser temperature T_c and the evaporator temperature T_e. The sum of the first two terms of Eq. (2) corresponds to the savings due to the replacement of the MDC by the TDC. The third term corresponds to the capital cost of the solar heating system, while the last term corresponds to the difference of the capital cost related to the TDC and the capital cost of the MDC. Here $\Delta C_{CT} = C_{CTTH} - C_{CTEL}$, where C_{CTTH} is the capital cost of the cooling tower required for the TDC and C_{CTEL} is the capital cost of the cooling tower required for the MDC. The size of the cooling tower depends strongly on the COP of the cycle.

The factor P_1 is the present worth factor of the series of annual savings due to the operating cost, while P_2 is the economical factor that takes into account the present value of the interest rate of the loan, income and property taxes, mortgage costs, resale value and depreciation costs.

The maximum of LCS is found by putting the partial derivative of LCS with respect to A_c to vanish. The result in terms of the derivative of f can be written as follows

$$\alpha_F \frac{\partial f}{\partial a_c} = \mathrm{COP} \tag{3}$$

where $a_c = A_c/Q_r$ and $\alpha_F = (P_1 C_{F1})/(P_2 C_A)$.

For ejector cycles, COP is an increasing function of T_g. On the other hand, f is a function of Q_L and therefore a function of COP. On the other hand it is shown that f as given in Klein and Beckman (1979) is an increasing function with the generator temperature T_g

and therefore, it makes sense to find the condition for which LCS reaches a maximum in T_g. By imposing the partial derivative of LCS with respect to T_g to vanish it follows

$$\frac{1}{(1-f)}\frac{\partial f}{\partial T_g}+\frac{1}{\mathrm{COP}}\frac{\partial \mathrm{COP}}{\partial T_g}=0 \tag{4}$$

For the particular case of the Carnot cycle, COP is shown in Sokolov and Hershgal (1993) to be given by

$$\mathrm{COP}=\frac{T_e}{T_g}\frac{(T_g-T_c)}{(T_c-T_e)} \tag{5}$$

Replacing COP from Eq. (5) in Eq. (4) it follows

$$\frac{1}{(1-f)}\frac{\partial f}{\partial T_g}+\frac{T_c}{T_g}\frac{1}{(T_g-T_c)}=0 \tag{6}$$

A particular case of practical interest is the case for which the life cost savings is assumed to vanish. By replacing α_F from Eq. (3) into Eq. (2) this limiting condition can be expressed by the following

$$(a_{\max}-a_c)\frac{\partial f}{\partial a_c}=1-f \tag{7}$$

where

$a_{\max}=\alpha_E+d/C_A$, $\alpha_E=P_1C_{E1}/(P_2C_A\mathrm{COP}_{el})$, and $d=(C_{EL}-C_{TH}-C_E-\Delta C_{CT})/Q_r$.

The specific area $a_{\max}$ is an upper bound for the specific area a_c, for all cases of non negative life cost savings, for the period of the economical analysis considered.

According to Klein and Beckman (1979) the fraction f is expressed as follows

$$f=\sum_{i=1}^{12}f_iQ_{L_i}/Q_L \tag{8}$$

By replacing Q_L as a function of Q_r given by Eq. (1) for both the annual and monthly basis it follows

$$f=\sum_{i=1}^{12}f_iQ_{r_i}/Q_r \tag{9}$$

where Q_{r_i} is the cooling demand and f_i is the solar fraction for month (i), the later expressed by the following correlation given in Duffie and Beckman (1991),

$$f_i=\bar{\phi}_{\max,i}Y_i-0.015(e^{3.85f_i}-1)(1-e^{-0.15X_i'})(R_S)^{0.76} \tag{10}$$

The parameters X_i' and Y_i, for the present analysis are modified in terms of COP as follows

$$Y_i=A_CF_R(\tau\alpha)_n\left[\frac{(\overline{\tau\alpha})}{(\tau\alpha)_n}\right]_i\overline{H}_{T_i}N_iP\mathrm{COP}/Q_{r_i} \tag{11}$$

$$X_i'=A_C(F_RU_L)100\Delta t_i\mathrm{COP}/Q_{r_i} \tag{12}$$

where $\Delta t_i=86400N_i$, N_i is the number of days of month (i); F_RU_L and $F_R(\tau\alpha)_n$ are the efficiency coefficients of the collector, $\overline{H}_{T_i}$ is the monthly average of the solar radiation incident on the tilted collector plate, and R_S is assumed here to be equal to the unity. The solar fraction f_i, as well as its derivatives with respect to A_c and T_g, are evaluated implicitly from Eq. (10) and given in Appendix A. The expression for the monthly average $\overline{H}_{T_i}$ estimated assuming isotropic sky is also given in Appendix B.

$$\bar{\phi}_{\max,i}=\bar{\phi}(\overline{X}_{c\min,i}) \tag{13}$$

$$\bar{\phi}(\overline{X}_c)=\exp\left[(a+bR_n/\overline{R})(\overline{X}_c+c\overline{X}_c^2)\right] \tag{14}$$

where $\bar{\phi}$ is the monthly average daily collector utilizability, and a, b, and c are functions of the average clearness index $\overline{K}_T$ for each month (i).

$$\overline{X}_{c\min,i}=F_RU_L(T_{\min}-\overline{T}_{a_i})/F_R(\tau\alpha)_n\left[(\overline{\tau\alpha})/(\tau\alpha)_n\right]_i(r_{t,n}R_n\overline{H})_i \tag{15}$$

where $\overline{X}_c$ is the dimensionless average daily critical level of the solar collector, $\overline{T}_{a_i}$ is the average outdoor temperature for the month (i). The parameters $R_n/\overline{R}$, $r_{t,n}$, and the correlations for a, b and c given in Duffie and Beckman (1991) are presented in the Appendix B. The monthly means of global radiation incident on the horizontal surface $\overline{H}_i$ for the site chosen for the numerical example can be found in Duffie and Beckman (1991).

3. Numerical example and discussion of results

The numerical example chosen here is the same example given in Klein and Beckman (1979) for the location of Albuquerque, New Mexico. The cooling capacity is taken to be 10.5 kW for 12 h of operation each day of the year. The minimum temperature $T_{\min}$ for the absorption cycle is 77 °C. The flat plate collector efficient coefficient are $F_R(\tau\alpha)_n=0.74$ and $F_RU_L=3$ $\mathrm{W\,m^{-2}\,K^{-1}}$. The COP for the lithium-bromide absorption system as given in Klein and Beckman (1979) is equal to 0.65. For the ejector cycle proposed here COP is 0.35, as found from data of Cizungu et al. (1999) for $T_{\min}=T_c=27.7$ °C, $T_g=77$ °C and $T_e=8.8$ °C.

Fig. 2 illustrates a solution for an optimum value a_c, for the absorption cycle with COP equal to 0.65. In this figure $h=(a_{\max}-a_c)\partial f/\partial a_c$ is plotted as a function of a_c. It is seen that function h vanishes in the point where $a_c=a_{\max}$.

Fig. 3 shows curves of LCS = 0 for the Carnot cycle case, the absorption case considered here and the case of the ejector. Points on the left and above the curve of a given LCS constant correspond to the economically

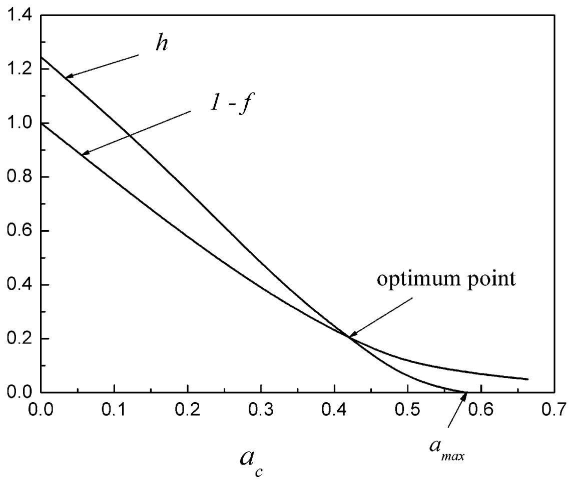

Fig. 2. Optimum solution for a_c, for the absorption cycle for $T_g = 77$ °C.

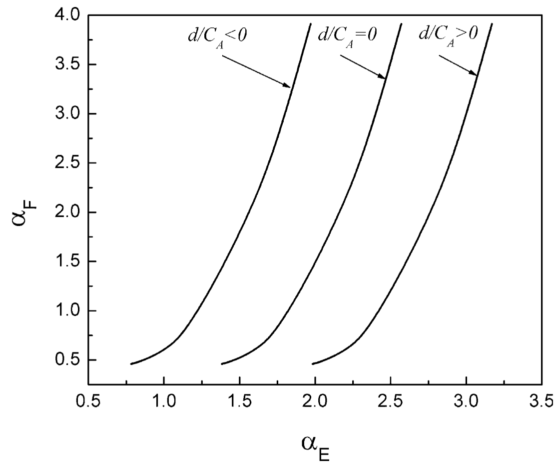

Fig. 4. Curves of LCS = 0 for constant d/C_A, for the absorption cycle.

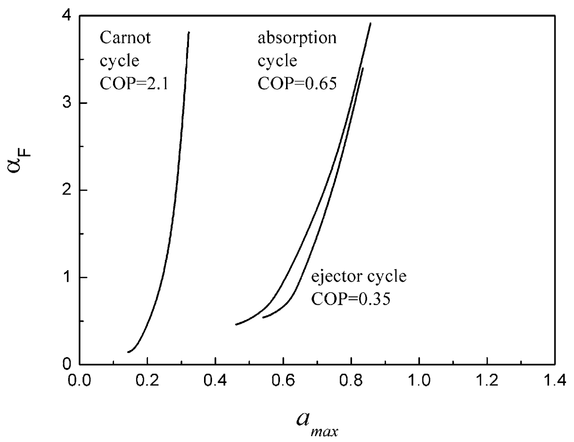

Fig. 3. Bound curves of LCS = 0, for the absorption cycle with $T_{min} = T_g = 77$ °C, and for the ejector cycle with $T_{min} = T_c = 27.7$ °C.

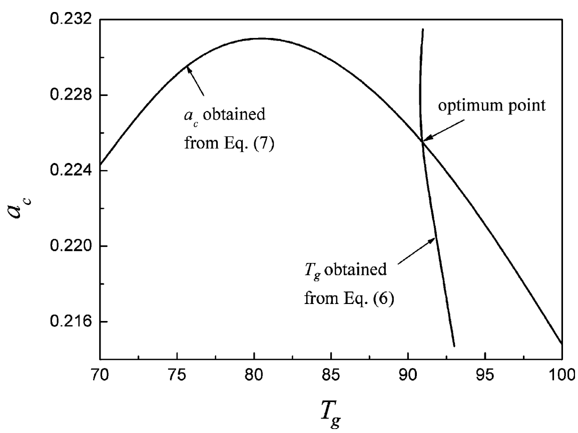

Fig. 5. Optimum solution for T_g and a_c for the case of the Carnot cycle.

unfeasible regions. It can be seen from this figure that the lesser the value of the performance coefficient the greater the required electricity cost in order to reach a feasible point, as expected.

Fig. 4 shows the curves of LCS = 0 as a function of the cost parameters α_E and α_F, for different values of the capital cost ratio d/C_A. Fig. 5 illustrates a solution for an optimum value of T_g and a_c, for the particular case of the Carnot cycle. This figure is obtained from solutions of Eq. (7) in terms of a_c, for given values of T_g, and solutions of Eq. (6) in terms of T_g, for given values of a_c. Fig. 6 shows the curves of LCS = 0 given in Fig. 3, which are plotted as functions of α_F and α_E, where $\alpha_E = a_{max} - d/C_A$. For the present numerical example the absorption chiller chosen is manufactured by Yazaki Co. Japan. The capital cost of the chiller is quoted around US\$5000.00 by the manufacturer. The capital cost of the ejector chiller is assumed to be US\$2000.00, for the same cooling capacity of 10.5 kW. This cost is around twice the capital cost of an MDC commercially available in the market.

As can be seen from Fig. 6, the economical feasible region corresponding to the ejector cycle is larger than the feasible region corresponding to the absorption cycle. The bound curve of the absorption cycle is shifted to the right by a difference between the capital cost related to the absorption system and the capital cost related to the ejector system. This advantage of the ejector cycle can be explained by the relationship between a_{max} and the costs considered.

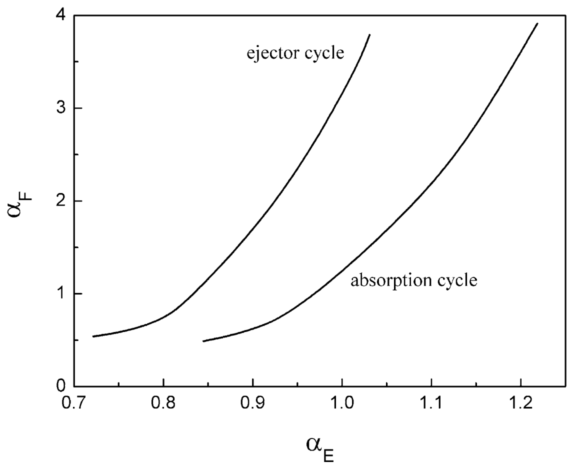

Fig. 6. Bound curves of LCS = 0 as a function of α_E for particular values of the capital cost.

From the definition of a_{max}, it can be seen that

$$\alpha_{Eabs} = \alpha_{Eej} + a_{max,abs} - a_{max,ej} + (C_{CTabs} - C_{CTej} + C_{THabs} - C_{THej})/Q_r C_A \quad (16)$$

where $C_{THabs} - C_{THej} =$ US\$3000.00. Here the capital cost of the cooling tower for both systems are assumed to be the same, i.e. $C_{CTabs} - C_{CTej} = 0$. Since the capital cost of the cooling tower corresponding to the ejector chiller is expected to be greater than the capital cost of the cooling tower corresponding to the absorption chiller, the later difference is expected to be negative, and therefore it may compensate the benefits due to the positive difference of the capital cost of the chillers themselves.

4. Conclusions

A design method for economical evaluation and optimization of thermally driven cooling cycles assisted by solar energy is presented. The analysis presented in this paper can be applied to determine the conditions under which the ejector cycle may be economically competitive with the absorption cycle, with no need of a full scale simulation of the related cooling system. The upper bounds for the region of feasibility of economical optimization of both, the ejector cycle and the absorption cycle are determined, in terms of the electricity cost and the auxiliary energy cost. It is shown that the cost of the chiller, as well as the cost of the respective cooling tower is of major importance, in setting down the conditions under which the ejector cycle become competitive with the absorption cycle. In favor of the absorption chiller is it's higher coefficient of performance, for the same type of flat plate collector. The expected lower value of the capital cost of the ejector chiller itself, in relation to the absorption chiller, is in favor of the economical advantage of the ejector cycle. However this economical advantage may decrease because of the need of greater size of the cooling tower for the ejector cycle.

Acknowledgements

The authors are indebted to CNPq-National Council of Research of Brazil/Ministry of Science and Technology for the support to the present work.

Appendix A. Determination of $\overline{H}_T$ and R_n

All the equations presented here for the determination of $\overline{H}_T$ and R_n for each month, are described in Duffie and Beckman (1991).

(a) Determination of $\overline{H}_T$

$$\overline{H}_T = \overline{R} \cdot \overline{H} \quad (A.1)$$

where $\overline{H}$ is the monthly average daily total solar radiation on a horizontal surface for Albuquerque, given in Duffie and Beckman (1991).

In Eq. (A.1), the ratio of the daily total radiation on a tilted surface to that on a horizontal surface, by assuming the isotropic sky model is expressed by

$$\overline{R} = \frac{\overline{H}_T}{\overline{H}} = \left(1 - \frac{\overline{H}_d}{\overline{H}}\right)\overline{R}_b + \frac{\overline{H}_d}{\overline{H}}\left(\frac{1+\cos\beta}{2}\right) + \rho_g\left(\frac{1-\cos\beta}{2}\right) \quad (A.2)$$

where $\overline{R}_b$ is given by

$$\overline{R}_b = \frac{\cos(\phi-\beta)\cos\delta\,\mathrm{sen}\,w'_S + (\pi/180)w'_S\,\mathrm{sen}(\phi-\beta)\,\mathrm{sen}\delta}{\cos\phi\cos\delta\,\mathrm{sen}\,w_S + (\pi/180)w_S\,\mathrm{sen}\,\phi\,\mathrm{sen}\delta} \quad (A.3)$$

$$\delta = 23.45\,\mathrm{sen}\left(360\frac{284+na}{365}\right) \quad (A.4)$$

where na, ϕ, β are known and w_S, w'_S are given by

$$\cos w_S = -tg\phi tg\delta \quad (A.5)$$

$$w'_S = \min\begin{bmatrix}\arccos(-tg\phi tg\delta)\\ \arccos(-tg(\phi-\beta)tg\delta)\end{bmatrix} \quad (A.6)$$

The ratio of the monthly average of the daily total of diffuse radiation, to the monthly average of the daily total of the global horizontal radiation is given by

For $w_S \leqslant 81.4°$ and $0.3 \leqslant \overline{K}_T \leqslant 0.8$

$$\frac{\overline{H}_d}{\overline{H}} = 1.391 - 3.560\overline{K}_T + 4.189\overline{K}_T^2 - 2.137\overline{K}_T^3 \quad (A.7)$$

For $w_S > 81.4°$ and $0.3 \leqslant \overline{K}_T \leqslant 0.8$

$$\frac{\overline{H}_d}{\overline{H}} = 1.311 - 3.022\overline{K}_T + 3.427\overline{K}_T^2 - 1.821\overline{K}_T^3 \quad (A.8)$$

where $\overline{K}_T = \overline{H}/\overline{H}_o$ and

$$H_o = \frac{24 \times 3600 G_{SC}}{\pi}\left(1 + 0.033\cos\frac{360na}{365}\right) \times \left(\cos\phi\cos\delta\,\mathrm{sen}\,w_S + \frac{\pi}{180}w_S\,\mathrm{sen}\,\phi\,\mathrm{sen}\delta\right) \quad (A.9)$$

(b) Determination of R_n

$$R_n = \left(\frac{I_T}{I}\right)_n = \left(1 - \frac{r_{d,n}}{r_{t,n}}\frac{H_d}{H}\right)R_{b,n} + \left(\frac{r_{d,n}}{r_{t,n}}\frac{H_d}{H}\right)\left(\frac{1+\cos\beta}{2}\right) + \rho_g\left(\frac{1-\cos\beta}{2}\right) \quad (A.10)$$

The ratio of the daily total of the diffuse solar radiation, to the daily total of the global radiation to the horizontal surface is given as follows

For $w_S < 81.4°$

$$\frac{H_d}{H} = \begin{cases} 1 - 0.2727K_T + 2.4495K_T^2 - 11.9514K_T^3 + 9.3879K_T^4 & \text{if } K_T < 0.715 \\ 0.143 & \text{if } K_T \geqslant 0.715 \end{cases} \quad (A.11)$$

For $w_S \geqslant 81.4°$

$$\frac{H_d}{H} = \begin{cases} 1 + 0.2832K_T - 2.5557K_T^2 + 0.8448K_T^3 & \text{if } K_T < 0.722 \\ 0.175 & \text{if } K_T \geqslant 0.722 \end{cases} \quad (A.12)$$

$$\frac{\partial f}{\partial T_g} = \frac{\bar{\phi}_{max}\partial Y/\partial T_g + Y\partial\bar{\phi}_{max}/\partial T_g - 0.00225(e^{3.85f} - 1)e^{-0.15X'}(R_S)^{0.76}\partial X'/\partial T_g}{1 + 0.05775(1 - e^{-0.15X'})e^{3.85f}(R_S)^{0.76}} \quad (B.4)$$

where $K_T = H/H_o$, and H_o is given by Eq. (A.9).

The ratio of beam radiation on the tilted surface to that on a horizontal surface at solar noon $R_{b,n}$ is expressed by

$$R_{b,n} = \frac{\cos|\phi - \delta - \beta|}{\cos|\phi - \delta|} \quad (A.13)$$

where ϕ, δ, and β are known.

On the other hand, $r_{d,n}$ and $r_{t,n}$ are given for the following expressions

$$r_{d,n} = \frac{\pi}{24}\frac{\cos w - \cos w_S}{\mathrm{sen}\,w_S - \frac{\pi}{180}w_S\cos w_S} \quad (A.14)$$

$$r_{t,n} = \frac{\pi}{24}(a + b\cos w)\left[\frac{\cos w - \cos w_S}{\mathrm{sen}\,w_S - \frac{\pi}{180}w_S\cos w_S}\right] \quad (A.15)$$

where

$$a = 0.409 + 0.5016\,\mathrm{sen}(w_S - 60)$$
$$b = 0.6609 - 0.4767\,\mathrm{sen}(w_S - 60)$$

Appendix B. Derivatives of the solar fraction *f*

The following derivatives are obtained from the Eq. (5) and Eqs. (10)–(15).

(a) The derivative of the solar fraction with respect to specific collector area is given by

$$\frac{\partial f}{\partial a_c} = \frac{\bar{\phi}_{max}\partial Y/\partial a_c - 0.00225(e^{3.85f} - 1)e^{-0.15X'}(R_S)^{0.76}\partial X'/\partial a_c}{1 + 0.05775(1 - e^{-0.15X'})e^{3.85f}(R_S)^{0.76}} \quad (B.1)$$

The derivatives in the above equation are calculated for each month as follows

$$\partial Y/\partial a_c = F_R(\overline{\tau\alpha})\overline{H}_T N\,\mathrm{COP} \quad (B.2)$$

and

$$\partial X'/\partial a_c = F_R U_L(100)\Delta t\,\mathrm{COP} \quad (B.3)$$

(b) The derivative of the solar fraction with respect to the generator temperature is given by

where

$$\partial Y/\partial T_g = a_c F_R(\overline{\tau\alpha})\overline{H}_T N\frac{\partial \mathrm{COP}}{\partial T_g} \quad (B.5)$$

$$\partial X'/\partial T_g = a_c F_R U_L(100)\Delta t\frac{\partial \mathrm{COP}}{\partial T_g} \quad (B.6)$$

$$\partial\bar{\phi}_{max}/\partial T_g = \frac{\partial\bar{\phi}_{max}}{\partial\overline{X}_c}\frac{\partial\overline{X}_{c\,min}}{\partial T_g} \quad (B.7)$$

where

$$\frac{\partial\bar{\phi}_{\max}}{\partial\overline{X}_{\mathrm{c\,min}}} = \left(a + b\frac{R_{\mathrm{n}}}{\overline{R}}\right)(1 + 2c\overline{X}_{\mathrm{c\,min}}) \times \exp\left[\left(a + b\frac{R_{\mathrm{n}}}{\overline{R}}\right)\left(\overline{X}_{\mathrm{c\,min}} + c\overline{X}^2_{\mathrm{c\,min}}\right)\right] \quad (B.8)$$

$$\frac{\partial\overline{X}_{\mathrm{c\,min}}}{\partial T_{\mathrm{g}}} = \frac{F_{\mathrm{R}}U_{\mathrm{L}}}{F_{\mathrm{R}}(\overline{\tau\alpha})r_{\mathrm{t,n}}R_{\mathrm{n}}\overline{H}} \quad (B.9)$$

and

$$a = 2.943 - 9.271\overline{K}_{\mathrm{T}} + 4.031\overline{K}^2_{\mathrm{T}} \quad (B.10)$$

$$b = -4.345 + 8.853\overline{K}_{\mathrm{T}} - 3.602\overline{K}^2_{\mathrm{T}} \quad (B.11)$$

$$c = -0.17 - 0.306\overline{K}_{\mathrm{T}} + 2.936\overline{K}^2_{\mathrm{T}} \quad (B.12)$$

The evaluation of $\partial f_i/\partial a_{\mathrm{c}}$ and $\partial f_i/\partial T_{\mathrm{g}}$ require the numerical value of f_i, which is obtained implicitly from Eq. (10).

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