Available online at www.sciencedirect.com

ScienceDirect

Solar Energy 80 (2006) 1453–1462

SOLAR ENERGY

www.elsevier.com/locate/solener

Uncertainty calculation applied to different regression methods in the quasi-dynamic collector test

M.G. Kratzenberg [a,*], H.G. Beyer [b], S. Colle [a]

[a] *Solar Energy Laboratory, Federal University of Santa Catarina, Department of Mechanical Engineering, Post Box 476, 88040-900 Florianópolis, SC, Brazil*

[b] *Institute of Electrical Engineering, University of Applied Sciences Magdeburg-Stendal (FH), 39114 Magdeburg, Breitscheidstrasse 2, Germany*

Received 15 November 2004; received in revised form 16 March 2006; accepted 24 March 2006
Available online 26 July 2006

Communicated by: Associate Editors Klaus Vajen and Ulrike Jordan

Abstract

This paper discusses and evaluates the quasi-dynamic test procedure for solar thermal collectors according to EN12975 in view of the uncorrelated regression uncertainty of the collector model parameters obtained from the test. In this discussion the results of two procedures for the regression coefficients identification, the least square (LS) and the weighted least square (WLS) regression methods are analyzed. The uncertainties of the both, the LS and WLS results are calculated and validated. The inter-comparison of the two methods shows comparable normalized efficiency curves, but superior performance of the WLS method with respect to the uncertainties.

Keywords: Quasi-dynamic collector test; Weighted least square regression; Confidence intervals

1. Introduction

Testing the efficiency of solar collectors is a basic pre-requisite to obtain performance characteristics for thermal solar collectors.

In order to quantify those characteristics, we must run a test to determine the parameters that determine the efficiency curve of those solar collectors.

In an international context, standards for the respective test procedures are given by EN12975 (CEN, 1998) and ISO9806 (ISO, 1994). In Brazil the standard is given by ABNT (1988). Only the EN12975 provides three different test procedures, the steady-state test (SST) under indoor and outdoor conditions and the quasi-dynamic test (QDT) under outdoor conditions. The QDT has the advantage of allowing the execution of more collector tests within the same time period, using the same test equipment and the same test facility as compared to the steady-state collector test under outdoor conditions (Kratzenberg et al., 2002). On the other hand the quasi-dynamic test requires a

* Corresponding author. Tel.: +55 48 331 9779; fax: +55 48 331 7615.
E-mail address: manfred@labsolar.ufsc.br (M.G. Kratzenberg).

0038-092X/$ - see front matter
doi:10.1016/j.solener.2006.03.010

Nomenclature

a_j	regression coefficient [different units]
b_0	incident angle modifier coefficient [unitless]
c_p	effective heat capacity of the fluid [J/(kg K)]
D_f	diffuse fraction [unitless]
error	measured minus modeled collector efficiency [–]
G	global solar irradiance [W/m^2]
G_d	diffuse solar irradiance [W/m^2]
G_b	beam solar irradiance [W/m^2]
k_1	linear heat loss coefficient [W/(m^{2x} K)]
k_2	quadratic heat loss coefficient [W/(m^2 K^2)]
k_3	effective thermal collector capacitance [J/(m^2 K)]
$K_{\theta b}(\theta)$	incidence angle modifier for direct radiation [unitless]
$K_{\theta d}$	incidence angle modifier for diffuse radiation [unitless]
$\dot{m}$	mass flow [kg/s]
T_{in}	inlet temperature [°C]
T_{out}	outlet temperature [°C]
T_m	mean collector temperature [°C]
T_a	ambient temperature [°C]
u	standard uncertainty [different units]
U	expanded uncertainty [different units]
var(a_j)	variance of the collector coefficient (corresponds also to the squared standard error s_e^2 of a_j) [different units]
$X_{i,j}$	regression variable [different units]
$(1-\alpha)$	significance for the statistical tests and the uncertainty estimates [unitless]
ΔT	difference between collector mean and ambient temperature [K]
η_{me}	measured efficiency value [unitless]
η_{mo}	modeled efficiency value [unitless]
η_0	zero loss efficiency at normal incidence [unitless]
η_{0_norm}	η_0 of the QDT normalized to the SST conditions [unitless]
θ	incident angle of the beam radiation [°]
$E(\sigma^2)$	estimation of the residual mean square error [W/m^2]^2

somewhat more demanding effort for the calculation of the collector coefficients.

There has already been made a comparison between SST and QDT (Fischer et al., 2004), but there have been no statements about the uncertainties. The purpose of the present work is to discuss and evaluate the test procedures in view of the uncertainty of the determination of the collector coefficients for the QDT. The least square – LS (Montgomery and Peck, 1992; Hoffmann and Vieira, 1987; ISO, 1995) and the weighted least square – WLS (Press et al., 1996) regression methods applied in the trend setting QDT (CEN, 1998; Kratzenberg et al., 2003, 2004) are presented in this paper. Uncertainties of the LS and WLS results are calculated. A methodology for the verification of real confidence limit of the LS and WLS uncertainties is presented.

2. Collector test rig

Fig. 1 shows the schematic of a collector test rig, used for outdoor collector tests, configured to perform both, steady-state and quasi-dynamic collector tests (EN12975) under outdoor conditions. According to EN12975 the collector has to be mounted with a collector tilt angle β of 45°.

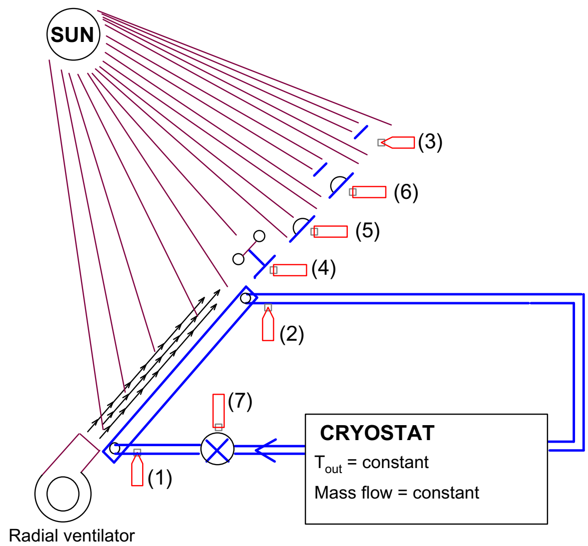

Fig. 1. Schematic diagram of the test rig for the quasi-dynamic and the steady-state collector test.

The following quantities are measured continuously: (1) inlet temperature: temperature of the fluid flowing into the collector, (2) outlet temperature: temperature of the fluid leaving the collector, (3) ambient temperature, (4) air speed: speed of the air at the collector front cover, (5) global irradiance: total solar irradiance measured in the tilted collector plane, (6) diffuse irradiance measured in the collector plane and (7) mass flow: measurement of the mass flow rate through the collector.

According to EN12975 the sampling rate has to be (1–6) s and an averaging period of 5–10 min for the mean values of the measured quantities has to be used in the analysis.

3. Measuring uncertainties

The transducers used for the measurements are selected according to the requirements on the uncertainty. The transducer uncertainty and the uncertainties of synchronization time of the measurement system that are used in the test are specified in Table 1.

The transducers were multiplexed with a measuring interval of 20 s and then computed the mean value of 5 min that were used for the regression.

4. Test conditions

SST and QDT specify different requirements to the weather conditions during the test, whereby the QDT test conditions are more similar to real collector operation conditions, i.e. with diffuse fraction $Df_{QDT} = (0–0.5)$ instead of $Df_{SST} = (0–0.3)$, global radiation $G_{QDT} = (300–1100)\ W/m^2$ instead of $G_{SST} = (700–1000)\ W/m^2$, incident angle $\theta_{QDT} = (0–60)°$ instead of $\theta_{SST} = (0–30)°$ and no limitation of the solar radiation variability in the quasi-dynamic-test instead of maximal variation $\pm 50\ W/m^2$ during the steady-state test.

Table 1
Ranges and uncertainties of the collector test variables

Parameter	Range	Uncertainty
Input temperature	10–100 °C	±0.1 °C
Output temperature	10–100 °C	±0.1 °C
Ambient temperature	10–50 °C	±1 °C
Global radiation	300–1100 W/m^2 *	±50 W/m^2 #
Diffuse radiation	0–800 W/m^2	±50 W/m^2 #
Mass flow	0.02 kg/(m^2 s)	±1%
Air speed over the collector aperture	1–6 m/s	±0.5 m/s
Collector tilt angle	40–50°	±1°
Measuring interval	1–6 s	±1%

CEN (1998) is not specifying these uncertainties explicitly; ISO9806 specifies the uncertainty with ±50 W/m^2 of the secondary standard pyranometer.
* CEN (1998) is not specifying the lower limit of 300 W/m^2, compare also CEN (2003) and Fischer et al. (2004).

The collector test discussed here was performed over the period of approximately one week. During this period sufficient data were acquired to perform the evaluation of the quasi-dynamic tests.

The data sub-set used for the extraction of the collector parameters were selected according to the reference conditions provided by EN12975 (CEN, 1998) which are:

- wind speed (2–4) m/s;
- $(T_{out} - T_{in}) > 1$ K, where T_{in} is the inlet temperature and T_{out} is the outlet temperature;
- dT_m/dt higher than ±0.005 K/s for some mean values for *semi covered sky condition* (see Eq. (1)), where dT_m is the 5 min mean value of the differences of the mean collector temperature between $T_{m,i}$ and $T_{m,i-1}$ and dt specifies the measuring interval between t_i and t_{i-1};
- only selection of values with positive power balance (positive η-values);
- stable mass flow of ±1% during a test day or test sequence;
- stable mass flow of ±5% during the whole collector test;
- stable input temperature of ±1 K in a valid data period;
- $T_m = T_a$ (±3) K for the data set from the clear sky day, where T_m is the mean collector temperature and T_a is the ambient temperature.

In addition a further discrimination was performed by

- the exclusion of values that can generate errors caused by heat flow direction changes of the heat losses (with positive η-values). Only data with positive ΔT values are selected;
- the exclusion of values where diffuse radiation fraction is >50%.

5. Regression methods for the determination of collector coefficients

The Euronorm (CEN, 1998) proposes the *multiple linear regression* (*MLR*) for the QDT using the thermal power output as indicator for the equivalence of measurement and model. Following the practice described by Müller-Schöll and Frei

(2000) and Sabatelli et al. (2002) for the SST, in this paper the WLS regression is accomplished for the efficiency values, i.e. the measured and modeled power values divided by the global irradiance in the collector plane (see. Eq. (1)).

The six regression variables of the model and the measured efficiency have to be determined from the measured data. By the regression procedure, the six regression coefficients can be determined with the goal to minimize the squared differences (or errors) between the calculated and the modeled efficiencies of all data pairs. As the uncertainties of the primary measurement data (radiations, flow rate, temperatures) effect the calculation of the efficiency, a procedure would be useful, which considers appropriate weighting of a data pair in respect to the inherent uncertainty of that data pair.

$$\eta_{\mathrm{mo}} = \overbrace{\underbrace{\underbrace{\frac{\eta_0^* G_{\mathrm{b}}}{G}}_{\text{beam model}} - \underbrace{\frac{\eta_0^* b_0 G_{\mathrm{b}}}{G}\left(\frac{1}{\cos\theta} - 1\right)}_{\text{angle of beam model}}}_{\text{beam radiation model}=K\theta b(\theta)\eta_0^* Gb/G} + \underbrace{\frac{\eta_0^* K_{\theta\mathrm{d}} G_{\mathrm{d}}}{G}}_{\text{diffuse model}}}^{\text{optical properties}}$$

$$- \overbrace{k_1 \cdot \frac{\Delta T}{G} - k_2 \frac{(\Delta T)^2}{G}}^{\text{heat loss properties}} - \overbrace{\frac{k_3}{G}\frac{\mathrm{d}T_{\mathrm{m}}}{\mathrm{d}t}}^{\text{thermal capacity } p} \tag{1}$$

Regression variables 1–6 and the measured efficiency η_{me}:

$$1{:}\ G_{\mathrm{b}}/G = X_1,\ 2{:}\ \mathrm{IA}_{G_b}/G = X_2,\ 3{:}\ G_{\mathrm{d}}/G = X_3,$$

$$4{:}\ \Delta T/G = X_4,\ 5{:}\ \Delta T^2/G = X_5,\ 6{:}\ \partial T_{\mathrm{m}}/(\partial t\ G) = X_6,$$

$$7{:}\ \eta_{\mathrm{me}} = \frac{\overbrace{\dot{Q}_{\mathrm{m}}}^{\text{collector power}}}{\underbrace{GA}_{\text{solar power}}} = \frac{\dot{m}c_p(T_{\mathrm{out}} - T_{\mathrm{in}})}{GA};$$

η_{me} measured efficiency, η_{mo} modeled or calculated efficiency;
G_{b} beam radiation [W/m^2], G_{b} diffuse radiation [W/m^2], $G_{\mathrm{b}} = G - G_{\mathrm{d}}$;
T_{m} average collector temperature $= T_{\mathrm{m}} = (T_{\mathrm{in}} + T_{\mathrm{out}})/2$ [°C], $\Delta T = T_{\mathrm{m}} - T_{\mathrm{a}}$;
T_{a} ambient temperature [°C], θ incidence angle [°].

Regression coefficients a_1–a_6 obtained from the regression:

1. $\eta_0^* = a_1$ zero loss efficiency [unitless];
2. $\eta_0^* b_0 = a_2$, b_0 factor to determine the incident angle modifier of the beam irradiance [unitless];
3. $\eta_0^* K_{\theta\mathrm{d}} = a_3$, $K_{\theta\mathrm{d}}$ incident angle modifier for diffuse radiation [unitless];
4. $k_1 = a_4$ heat loss coefficient [W/(m^2 K)];
5. $k_2 = a_5$ heat loss coefficient [W/(m^2 K)];
6. $k_3 = a_6$ coefficient for the thermal capacity [kJ/(m^2 K)].

For example, pyranometers generally show a higher relative measuring uncertainty in their lower measurement range than in their upper range. The *weighted least square method* (*WLS*) weights measuring points at lower radiation (300 W/m^2) less than those at higher radiation (1100 W/m^2) within the regression process. Articles (Mathioulakis et al., 1999; Müller-Schöll and Frei, 2000; Sabatelli et al., 2002) remark that the *MLR regression* based on the *least square method* (*LS*) is not most efficient to determine the collector coefficients and their uncertainties for the SST, as stated by Mathioulakis et al. (1999): '*The problem with this method is that, in reality, the typical deviation σ is almost never constant and the same for all points, but each data point $\eta_{\mathrm{me},i}, x_i$ has its own standard deviation σ_i*'. This paper shows the advantage of *WLS* applied for the QDT. In the following we discuss the different results obtained by the *LS* and the *WLS* for the QDT.

6. The weighted least square method and the least square method used on the QDT

6.1. Calculation of the collector coefficients

WLS is a weighted regression method, as the name says. The method is used for collector coefficients and uncertainties calculations. Theoretical background of this method is given by Press et al. (1996). Compared to *LS* it has the following advantages:

- the collector coefficients are determined also using the measurement uncertainties of the transducers;
- the measurement uncertainty of the transducer can be different in different measurement ranges;
- the statistic distribution of the measurement uncertainties does not have to be normally distributed;
- the measurement uncertainties may be weakly correlated with each other.

Eq. (2) gives the individual squared errors for each 5 min mean value set (counted by $i = 1$–n) as function of the collector coefficients a_1–a_6 (counted by $j = 1$–k) and the measured variables $X_{1,i}$–$X_{6,i}$ calculated with the data from the measurement (see Eq. (1)).

$$\text{error}_i^2 = (\eta_{\text{me},i} - \eta_{\text{mo},i})^2 = \left(\eta_{\text{me},i} - \sum_{j=1}^{k}(X_{j,i} \cdot a_k)^2\right)^2 \quad (2)$$

In the next step, the uncertainties (Press et al., 1996) of all error-values, denominated $u_{\text{error},i}$ have to be calculated (Eq. (3)), where $u(X_{1,i})$ to $u(X_{6,i})$ (counted by $j = 1$–6) are the *standard uncertainties* (ISO, 1995) of the regression variables and $u(\eta_{\text{me},i})$ are the *standard uncertainties* of the measured efficiency values.

$$u_{\text{error},i}^2 = \left(\frac{\partial(\text{error}_i)}{\partial(\eta_{\text{me},i})} u(\eta_{\text{me},i})\right)^2 + \sum_{j=1}^{k}\left(\frac{\partial(\text{error}_i)}{\partial(X_{j,i})} u(X_{j,i})\right)^2 \quad (3)$$

The measurement uncertainties of the measured efficiency $u(\eta_{\text{me},i})$ and every variable $u(X_{1,i})$ to $u(X_{6,i})$ have to be determined taking into account the standard uncertainties of the transducers to calculate the standard uncertainty $u_{\text{error},i}$ (Eq. (3)). After the squared quotients of the error_i – and $u_{\text{error},i}$ – values have been determined, the new weighted evaluation function χ^2 can be implemented (Eq. (4)). In calculating χ^2, the individual errors are thus weighted by its reciprocal uncertainty. Within these calculation the coefficients a_1–a_6, determined by a *LS* regression can be used as initial values for the minimization of χ^2.

For the WLS, the collector coefficients are now varied iteratively by a spreadsheet program (e.g. EXCEL™) with the goal to minimize the sum of χ^2 for the n data points.

$$\chi^2 = \sum_{i=1}^{n} \frac{\text{error}_i^2}{u_{\text{error},i}^2} \to \min \quad (4)$$

The collector coefficients resulting form the two different regression methods (WLS and LS) are documented in the section of the results (Section 8, Table 2).

For the LS regression the coefficients are obtained by minimizing the sum of all squared error_i^2 – values given by Eq. (2).

6.2. Calculation of the collector coefficient uncertainties with WLS-method

For the determination of the uncertainties of collector coefficients in the framework of the *WLS regression*, the method described by Press et al. (1996) is applied here. The application of this method for the SST collector test is described in Müller-Schöll and Frei (2000); here it is used for the QDT collector test for determining the uncertainties for the collector coefficients gained by the *WLS regression*. The calculation of the uncertainties of the collector coefficients starts with determining a matrix A (Eq. (5)) using the variables $X_{j,i}$ from a data set of one collector test and the uncertainties $u_{\text{error},i}$ (3) of each 5 min mean value from the same data set. In Press et al. (1996), it is shown that from a 6×6 matrix $[C]$ (Eq. (6)), whose diagonal elements present the squared standard uncertainties (or variances) of the target parameters, can be determined the standard uncertainties of the regression coefficients. The off-diagonal elements give the co-variances between the regression coefficients.

$$[A] = \begin{pmatrix} \left\langle \frac{X_{1,1}}{u_{\text{error},1}} \right\rangle & \left\langle \frac{X_{2,1}}{u_{\text{error},1}} \right\rangle & \langle \cdots \rangle & \left\langle \frac{X_{6,1}}{u_{\text{error},1}} \right\rangle \\ \left\langle \frac{X_{1,2}}{u_{\text{error},2}} \right\rangle & \left\langle \frac{X_{2,2}}{u_{\text{error},2}} \right\rangle & \langle \cdots \rangle & \left\langle \frac{X_{6,2}}{u_{\text{error},2}} \right\rangle \\ \langle \cdots \rangle & \langle \cdots \rangle & \langle \cdots \rangle & \langle \cdots \rangle \\ \left\langle \frac{X_{1,n}}{u_{\text{error},n}} \right\rangle & \left\langle \frac{X_{2,n}}{u_{\text{error},n}} \right\rangle & \langle \cdots \rangle & \left\langle \frac{X_{6,n}}{u_{\text{error},n}} \right\rangle \end{pmatrix} \quad (5)$$

The variances that corresponds to the squared standard uncertainties of the regression coefficients are determined (Press et al., 1996) for the WLS regression with the following equation:

Table 2
Collector coefficients with uncertainties in the 95% confidence interval

	LS				WLS				Units
	min	coeff.	max	U	min	coeff.	max	U	
η_0	0.710	0.716	0.722	0.006	0.707	0.713	0.718	0.005	[unitless]
b_0	0.119	0.144	0.170	0.026	0.106	0.128	0.149	0.022	[unitless]
$K_{\theta d}$	0.827	0.868	0.908	0.041	0.856	0.894	0.933	0.038	[unitless]
k_1	6.445	5.890	5.335	0.555	6.532	6.109	5.686	0.423	[W/(m^2) K]
k_2	0.049	0.038	0.027	0.011	0.043	0.035	0.027	0.008	[W/(m^2) K]
C_{eff}	3821.2	636.0	2549.1	3185.1	3039.4	612.1	1815.2	2427.3	[KJ/(m^2) K]

$$[C] = [[A^{T}][A]]^{-1} = \begin{bmatrix} E[\mathrm{var}(\hat{a}_0)] & E[\mathrm{cov}(\hat{a}_0,\hat{a}_2)] & \cdots & E[\mathrm{cov}(\hat{a}_0,\hat{a}_k)] \\ E[\mathrm{cov}(\hat{a}_2,\hat{a}_0)] & E[\mathrm{var}(\hat{a}_2)] & \cdots & E[\mathrm{cov}(\hat{a}_2,\hat{a}_k)] \\ \vdots & \vdots & \ddots & \vdots \\ E[\mathrm{cov}(\hat{a}_k,\hat{a}_0)] & E[\mathrm{cov}(\hat{a}_k,\hat{a}_2)] & \cdots & E[\mathrm{var}(\hat{a}_k)] \end{bmatrix} \quad (6)$$

The expanded uncertainties (ISO, 1995), of the regression coefficients are calculated multiplying the square root of the variances with the student-t value, where $t_{\alpha/2,n-k}$ is the student-t value of the regression for $(1-\alpha)100\%$ of confidence with n mean values (5 min) and k regression coefficients.

6.3. Calculation of the collector coefficients uncertainties for the LS-method

In ISO (1995) the variances that correspond to the squared standard uncertainties of the regression coefficients are determined for the LS regression with the following equation:

$$\sigma^2[[X]^{T}[X]]^{-1} = \begin{bmatrix} E[\mathrm{var}(\hat{a}_0)] & E[\mathrm{cov}(\hat{a}_0,\hat{a}_2)] & \cdots & E[\mathrm{cov}(\hat{a}_0,\hat{a}_k)] \\ E[\mathrm{cov}(\hat{a}_2,\hat{a}_0)] & E[\mathrm{var}(\hat{a}_2)] & \cdots & E[\mathrm{cov}(\hat{a}_2,\hat{a}_k)] \\ \vdots & \vdots & \ddots & \vdots \\ E[\mathrm{cov}(\hat{a}_k,\hat{a}_0)] & E[\mathrm{cov}(\hat{a}_k,\hat{a}_2)] & \cdots & E[\mathrm{var}(\hat{a}_k)] \end{bmatrix} \quad (7)$$

where the matrix $[X]$ is given by n horizontal vectors (n is the number of data points used for the regression), each with the variables $X_{1,i}$ to $X_{k,i}$. It is the same matrix as in Eq. (5), but without taking into account the uncertainties.

The uncertainties of the collector coefficients determined for the results of the two different regression methods (WLS and LS) are documented in the section of the results (Section 8, Table 2).

6.4. Calculation of the uncertainties of the mean model response for the LS- and WLS regression method

The root of the variances $u^2_{\eta i}$ of the collector model response is calculated with Eq. (8) as recommended in ISO (1995) if no correlation of the regression coefficients is considered. $u_{\eta i}$ obtained for the two different regression methods (WLS and LS) are used to calculate the expanded uncertainties, multiplying them with the student-t value (ISO, 1995). In the section of the results (Section 8, Fig. 5) the expanded uncertainty intervals of the normalized efficiency curves are shown.

$$u^2_{\eta\mathrm{mo},i} = \sum_{j=1}^{k} \left(\frac{\partial(\eta_{\mathrm{mo},i})}{\partial(a_k)} u_{\mathrm{a},j} \right)^2 \quad (8)$$

7. Validation of the confidence intervals

Assuming the case that the measured efficiency η_{me} is always equal to the modeled efficiency η_{mo}, we would get a straight line as presented in Fig. 1 (identity line). To assess the significance of a devia-

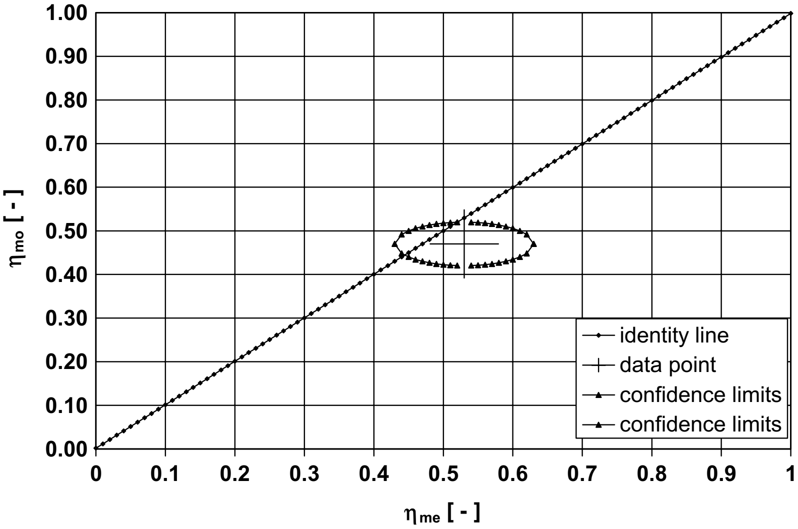

Fig. 2. Example of one data point with his uncertainties within a 95% confidence limit.

tion from this line (i.e. η_{me} and η_{mo} are not statistical identical) confidence limits (95%) according to the uncertainties of η_{me} and the prediction intervals of η_{mo} can be analyzed. Taking into account both, the uncertainties of η_{me} and the prediction intervals of η_{mo} (Montgomery and Peck, 1992) we get an *uncertainty-prediction ellipse* for each measured point (Fig. 2). Using a confidence limit of 95%, the ellipse indicates the boundaries in which the region within the 'real' point of η is located with a probability of 95%. Identity of measured and modeled efficiencies in this sense is given when the ellipse covers the identity line.

Using this presentation, the accuracy of the uncertainty estimates can be evaluated. Based on the given data set, it has to be checked whether the fraction of cases with *uncertainty – prediction interval ellipses* covering the identity line is similar to the confidence limit selected. For this purpose it is checked whether an intersection $\eta_{mo,IS,i}$ of the circumference of the *uncertainty – prediction interval ellipse* with the identity line exists (see Eq. (6)).

$$\eta_{mo,IS,i} = \eta_{mo,i} \pm \sqrt{\left(1 - \frac{\eta_{me,i}^2}{U^2(\eta_{me,i})}\right) PI^2(\eta_{mo,i})} \quad (9)$$

where $\eta_{me,i}$ and $\eta_{mo,i}$ are the measured and the modeled efficiencies, $U(\eta_{me,i})$ is the expanded uncertainty of the measured efficiency and $PI(\eta_{mo})$ is the prediction interval of the modeled efficiency and where the expanded uncertainty is the standard uncertainty multiplied with the student-t value of the chosen confidence of 95%.

In Montgomery and Peck (1992), Hoffmann and Vieira (1987) the authors specify the calculation of the prediction interval of the model response of the ith measured value for a multiple linear regression model using an determined confidence as given in Eq. (10).

$$PI(\eta_{mo,i}) = \pm t_{\alpha/2,n-k}\sqrt{var(D_i)} \quad (10)$$

where $t_{\alpha/2,n-k}$ is the student-t value for $(1-\alpha)100\%$ of confidence with $k = 6$ regression coefficients (Eq. (1)) and n mean values used for the regression. The value $var(D_i)$ is the variance of the difference between the measured and the modeled efficiency and is determined by

$$var(D_i) = var(\eta) + E(\sigma^2) \quad (11)$$

The estimate of the residual mean square error $E(\sigma^2)$, denominated also MSE, can be determined with the following equation:

$$s^2 = E(\sigma^2) = \frac{\sum_{i=1}^{N}(\epsilon_i)^2}{n-k} = \frac{\sum_{i=1}^{N}(\eta_{mo,i} - \eta_{me,i})^2}{n-k} \quad (12)$$

The variance of the mean model response $var(\eta)$, also denominated squared standard error in the LS-regression is determined by Montgomery and Peck (1992) and Hoffmann and Vieira (1987) with

$$var(\eta) = u(\eta)^2 = \sigma^2\{X_{i,0}\}[[X]^T[X]]^{-1}\{X_{i,0}\}^T \quad (13)$$

where the vector $\{X_{i,0}\}$ is one of the horizontal vectors of the matrix $[X]$ (Eq. (7)) at the time i for which the prediction interval is calculated from $i = (1–n)$ mean values. The matrix $[X]$ is the same matrix as in Eq. (7).

In the WLS regression the variances of the regression coefficients $var(a_k)$ are determined by the $(A^TA)^{-1}$ matrix (Eq. (6)) and are used here to calculate the variances of the model with Eq. (8).

The uncertainty of the ith measured efficiency is calculated by Eq. (14).

$$(u_{\eta me,i})^2 = \left(\frac{\partial(\eta_{me,i})}{\partial\dot{Q}_{me,i}} u_{\dot{Q}me,i}\right)^2 + \left(\frac{\partial(\eta_{me,i})}{\partial G_i} u_{G,i}\right)^2 + \left(\frac{\partial(\eta_{me,i})}{\partial A} u_A\right)^2 \quad (14)$$

$u_{\dot{Q}me,i}$ standard uncertainty of the collector power;
$u_{G,i}$ standard uncertainty of the global radiation;
u_A standard uncertainty of the collector area.

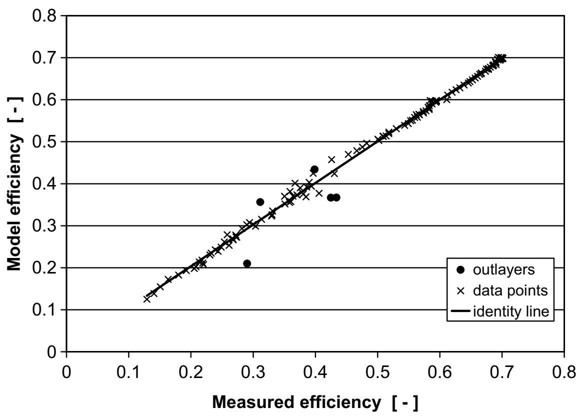

Fig. 3. Scatter diagram of measured and modeled efficiency values for the model stemming from the LS procedure. Data points with uncertainty ellipses overlapping the identity line are marked by crosses. Circles indicate the *out-layers*.

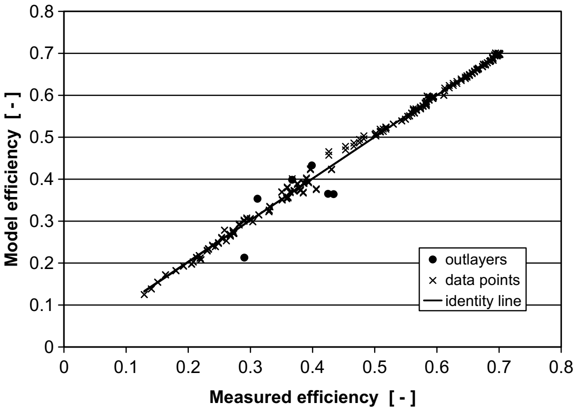

Fig. 4. Same as Fig. 2, but for model data from the WLS procedure.

For both Eqs. (3 and 14) transducer uncertainties as defined by EN12975 (Table 1) are used.

The points with non-intersecting uncertainty ellipsis for both *methods* are indicated in Figs. 3 and 4 for the uncertainties specified in Table 1. It can be stated that for the case of the LS data in about 4.8% of the cases (denominated out-layers) the *uncertainty – prediction interval ellipse* does not cover the identity line (expected value is 5%). For the WLS set these values amounts to about 6.5%. Using for the LS method Eq. (8) instead of the Eq. (13) to estimate the variance of the model response, we obtain 5.2% of *out-layers*.

8. Results

The expanded uncertainties of the regression coefficients and the mean response for the two regression methods are obtained by multiplying the standard uncertainties with the student-t values. Table 2 shows the collector coefficients determined by the *LS* and *WLS procedures* that are computed from the regression coefficients. Also given are the expanded uncertainties for a confidence interval of 95% and the respective upper and lower bounds for the coefficients provided.

For the table, the regression coefficients are transformed to collector coefficients using relations given in Eq. (1) and the uncertainties of the regression coefficients are transformed to the uncertainties of the collector coefficients using the combined uncertainty calculation assuming uncorrelated uncertainties (ISO, 1995).

With Eqs. (15) and (16) we can calculate the normalized efficiency curves as defined by EN12975 (CEN, 1998). These curves refer to the following conditions:

- beam radiation: 680 W/m^2 (85% of the global radiation);
- diffuse radiation: 120 W/m^2 (15% of the global radiation);
- global radiation: 800 W/m^2;
- Incidence angle: 15°.

The normalized efficiency curve is – for these conditions – defined by

$$\eta_{\text{norm}} = \eta_{0,\text{norm}} - k_1 \frac{T_\text{m} - T_\text{a}}{G} - k_2 \frac{(T_\text{m} - T_\text{a})^2}{G} \tag{15}$$

with $\eta_{0,\text{norm}}$ given by

$$\eta_{0,\text{norm}} = \eta_0 \left(\frac{G_\text{b}}{G} K_{\theta\text{b}}(15^\circ) + \frac{G_\text{d}}{G} K_{\theta\text{d}} \right);$$

$$K_{\theta\text{b}} = 1 - b_0 \left(\frac{1}{\cos\theta} - 1 \right) \tag{16}$$

The uncertainty of each point of the normalized efficiency curves (Fig. 5) are calculated using Eq. (8) for both regression methods (ISO, 1995).

9. Discussion of the results

Comparing the individual measured efficiencies to those calculated using the coefficient sets from both the *LS* and the *WLS method*, no considerable

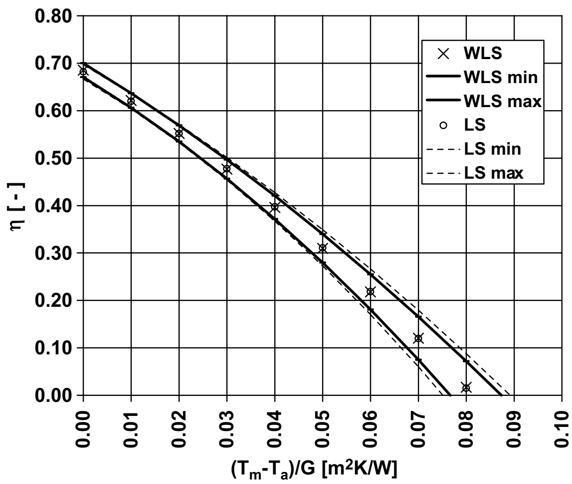

Fig. 5. Normalized efficiency curves from LS (diamonds) and WLS (crosses) data with the respective uncertainty limits. The limits refer to the 95% confidence interval.

deviations (errors) occur throughout most of the test data (Fig. 6). This holds for both methods.

In Fig. 6 we show the error values (applying the two different regression methods) between the modeled and the measured efficiency for the whole test data set. The curve denominated *delta models LS–WLS* presents the differences between the results of the two models (*WLS* and *LS*).

As the first 90 measuring points (Fig. 6) where taken with nearly *clear sky conditions*, it is most likely that the collector's *optical behavior* (determined by the coefficients η_0, b_0 and K_{0d}) were determined reliable. This is also shown by the small uncertainty values of these parameters (Table 2) and the small error values (Fig. 5).

In Fig. 5 the results of the *LS* and *WLS methods* are given by a normalized efficiency curve calculated with Eqs. (15) and (16).

From Table 2 we can see that the coefficient for the thermal capacity has a relatively high uncertainty. That uncertainty does not appear in the normalized efficiency curve (Fig. 4), because only the uncertainty of the non-dynamic coefficients $U(a_1)$ to $U(a_5)$ and variables $U(x_1)$ to $U(x_5)$ are used for the calculation of the total uncertainty with Eq. (11).

10. Conclusion

Fig. 4 and Table 2 show that the *weighted least square fitting method* has lower uncertainties than the *least square fitting method*. This proves the hypothesis that the *WLS method* leads to better regression results. In our example, the optical coefficients of the *WLS fit* do not differ much from the coefficients obtained from the *LS fit*. The small differences of the normalized efficiency curve are mainly a result of the differences in the heat loss coefficients from *WLS* as compared to the *LS*. We have to remark that the present results are only based on one collector test. In further collector tests with other collectors the differences of the coefficients may be higher. Anyway the *WLS method* is the method that delivers more adequate results for the quasi-dynamic test. The post-verification of the 95% confidence limit shows that the uncertainty calculations are correct, and gives more credit to the calculated uncertainty of the test results.

11. Summary and outlook

We calculate the collector coefficients and their uncertainties with the *least square* and the *weighed least square methods* for a quasi-dynamic test (CEN, 1998).

The 95% confidence limits for each collector coefficient are calculated as well as the respective limits for the collector efficiencies resulting from the identified model. Using the test data, the confidence limits for the efficiencies can be validated, which proves that the uncertainties of the *collector*

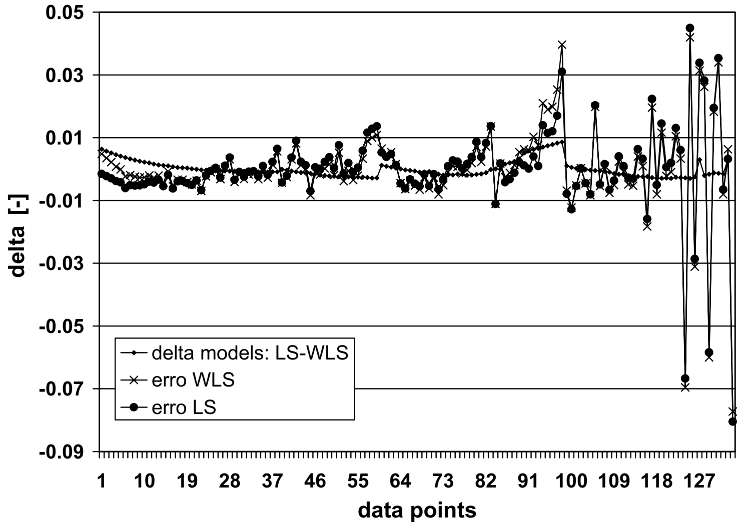

Fig. 6. Errors of the efficiency models calculated with the parameters sets from LS (circles) and WLS (crosses) procedures. The data for the 134 experimental values are sorted according to data number. The mutual deviation of the two models is given by the line indicated with small dots.

*coefficient*s as well as the *efficiency curve* of the collector can be determined in a reliable way. However, a review of these uncertainty statements still requires a more comprehensive analysis of several complete tests.

The *weighted least square method* (*WLS*) gets slightly different coefficients with the same collector as the *least square method* (*LS*).

The quasi-dynamic and the steady-state tests are both performed under outdoor conditions, and are thus, regarding the measurement conditions, impossible to repeat, as weather conditions (combination of solar radiation and ambient temperature) always vary. Therefore, to get a quantitative statement on the test reproducibility, the need for an extended database is again underlined. It should be remarked that this reproducibility is strictly related to the defined standard test conditions (or data selection conditions) as given by EN12975 (CEN, 1998).

The easy implementation in EXCEL™ spreadsheet makes it possible to apply the *WLS method* regularly for collector test evaluations.

References

ABNT, 1988. Standard NBR 10184 Coletores solares planos para líquidos – Determinação do rendimento térmico. Associação Brasileira de Normas Técnicas, Brazil.

CEN, 1998. Standard EN12975-2 Solar thermal systems and components – Solar collectors – Part 2: Test methods. CEN – European Committee for Standardization, Belgium.

CEN, 2003. CEN12975-1&2, Amendment – List of comments CEN/TC312/WG1N243, CEN – European committee for standardization, Belgium.

Fischer, S., Heidemann, W., Müller-Steinhagen, S., Perers, B., Bergquist, P., Hellström, B., 2004. Collector test method under quasi-dynamic conditions according to the European Standard EN 12975-2. J. Solar Energy 76, 117–123.

Hoffmann, R., Vieira, S., 1987. Análise de regressão – Uma introdução à econometria. Editora Hucitec, São Paulo, Brazil.

ISO, 1994. Standards 9806-1: Test Methods for Solar Collectors. Part 1: Thermal Performance of Liquid Heating Collectors Including Pressure Drop. International Organization for Standardization, Switzerland.

ISO, 1995. Guide to the Expression of Uncertainty in Measurements. International Organization for Standardization, Switzerland.

Kratzenberg, M., Beyer, H.G., Colle, S., 2002. Setup of a test facility for the characterization of thermal collectors according to the Euronorm at the Universidade Federal de Santa Catarina. In: Proceedings of Sun at the End of the World International Solar Energy Congress and Exhibition, Universidad Técnica Federico Santa María, Chile.

Kratzenberg, M., Beyer, H.G., Colle, S., Albertazzi Gonçalves, A., 2003. Test facility for quasi-dynamic collector tests for the characterization of thermal solar collectors in accordance with the international norms, metrologia. In: Proceedings of Metrologia para a Vida, Sociedade Brasileira de Metrologia (SBM), Recife, Brazil.

Kratzenberg, M., Beyer, H.G., Colle, S., Petzoldt, D., 2004. Bestimmung der Kollektorparameter und ihrer Unsicherheiten über die Methode der gewichteten Fehlerquadrate für den statischen und den quasi-dynamischen Kollektortest. In: Proceedings of Otti-Kolleg-Thermische Solarenergie, Staffelstein, Germany.

Mathioulakis, E., Voropoulos, K., Belessiotis, V., 1999. Assessment of uncertainty in solar collector modeling and testing. J. Solar Energy 66, 337–347.

Montgomery, D.C., Peck, E.A., 1992. Introduction to Linear Regression Analysis. Arizona State University, John Wiley & Sons, New York, USA, 527 p.

Müller-Schöll, C., Frei, U., 2000. Uncertainty analysis in solar collector measurements. In: Proceedings of the Eurosun Congress, Copenhagen, Denmark.

Press, W., Teukolsky, S.A., Vetterling, W.T., Flannery, B.P., 1996. Numerical Recipes, second ed. Cambridge University Press, Oxford.

Sabatelli, V., Marano, D., Braccio, G., Sharma, V.K., 2002. Efficiency test of solar collectors: uncertainty in the estimation of regression and sensitivity analyses. J. Energy Convers. Manage. 42.